KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

Marks Scored:

Level : B.Sc.
Year : II

Course : MATH 201
Semester : I

Exam Roll No. : Time: 30 mins. F. M. : 20

Registration No.: Date 3 0 JUN 2023

SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) with the most appropriate answer(s):

1. The radius of the circle $r = 4\cos\theta$ is ________________.

2. If $x = r\cos\theta,\ y = r\sin\theta$, then the jacobian $\frac{\partial(x,y)}{\partial(r,\theta)} =$ ____________________.

3. $\int_0^1 x\, dx^2 =$ ____________________.

4. If n is a positive integer, then $\Gamma(n+1) =$ ____________________, where the symbol has its usual meaning.

5. A stationary point that is not an extreme point is called a ____________________ point.

6. $\int_C (x+y)\, dx =$ ________________, where C is a straight line from the point $(1,0)$ to the point $(0,0)$.

7. $\int_{-2}^{0}\int_{y}^{-y} 2\, dx\, dy =$ ________________.

8. $\lim_{x\to 0} e^{-1/|x|} =$ ____________________.

9. The spherical coordinates (ρ, ϕ, θ) of the cylindrical coordinates $(\sqrt{2}, 0, 1)$ are ________________.

10. $\int_0^a \int_0^b \int_0^c dx\, dy\, dz =$ ______________________.

SECTION "B"

[10Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answers from among the given ones.

11. The Cartesian equation of the cylindrical equation $r = 3\sec\theta$ is ________________.
[$x = 3$, $x = -3$, $y = 3$, $y = -3$]

12. If $f(x,y) \in C^1$, then the directional derivative $\frac{\partial f}{\partial \xi_\alpha} =$ ______________________.
[$f\cos\alpha + f\sin\alpha$, $f_1\sin\alpha + f_2\cos\alpha$, $f_1\cos\alpha + f_2\sin\alpha$, $f_1 f_2 \sin 2\alpha$]

13. $\int_a^b d[x] =$ ____________________, where $[x]$ is the greatest integer function.
[$[b]-[a]$, $b-a$, $[a]-[b]$, $a-b$]

14. If $0 < m < 1$, then $\Gamma(m)\Gamma(1-m) =$ ____________________.
[$m!$, $\sin m\pi$, $\frac{1}{\sin m\pi}$, $\frac{\pi}{\sin m\pi}$]

15. Let $D \subset \mathbb{R}^2$ be a domain and $(a,b) \in D$. Let $f(x,y) \in C^2$ in D, and $f_1 = f_2 = 0$ at (a,b). Then f has a relative minimum at (a,b) if $f_{22} > 0$ and $f_{11}f_{22} - f_{12}f_{21}$ has a ____________________ value at (a,b).
[zero, negative,
positive, greater or equal to zero]

16. Let R be the region enclosed by a simple closed curve C in the plane oriented counterclockwise. Then the area of the plane region R is given by the integral ____________________.
[$\oint_C y\,dx - x\,dy$, $\frac{1}{2}\oint_C y\,dx - x\,dy$, $\oint_C x\,dy - y\,dx$, $\frac{1}{2}\oint_C x\,dy - y\,dx$]

17. The elementary volume $dV = dx\,dy\,dz$ in cylindrical coordinates (r,θ,z) is _________.
[$r\,d\theta\,dz$, $\theta\,dr\,dz$, $r\,\theta dz$, $r\,dr\,d\theta\,dz$]

18. If $u + \ln u = xy$, then $\frac{\partial u}{\partial x} =$ ____________________.
[$\frac{u}{u+1}$, $\frac{ux}{u+1}$, $\frac{uy}{u+1}$, $\frac{xy}{u+1}$]

19. An integral of the form $\int_a^b f(x)\,d\alpha(x)$ is known as ____________________ integral.
[Riemann, Legesgue, Stieltjes, Mobius]

20. $\int_0^1 x^{\frac{3}{2}}(1-x)dx =$ ____________________.
[$\frac{2}{35}$, $\frac{1}{35}$, $\frac{4}{35}$, $\frac{4}{25}$]

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3 0 JUN 2023

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F. M. : 55

SECTION "C
[3Q. × 7 = 21 marks]

1. a. Define the differentiability of a function f at (a, b) in $\mathbb{R}^2$. [2]

b. If $f(x,y) \in C^1$, then prove that $f(x,y)$ is differentiable at every point of domain $D \subseteq \mathbb{R}^2$. [4]

c. Show that $f(x,y) = x^2 + 2xy + y^2$ is differentiable in $\mathbb{R}^2$. [1]

OR

a. Prove that the transformation $u = f(x,y)$ and $v = g(x,y)$ with jacobian $J = \frac{\partial(u,v)}{\partial(x,y)} \neq 0$ have an inverse with jacobian $j = \frac{\partial(x,y)}{\partial(u,v)}$. [5]

b. Find $\frac{\partial u_{x,y}}{\partial x}$ if $u = f(x,y)$ and $y = g(x,y)$ where the symbol has its usual meaning. [2]

2. a. State the limit definition of the Stieltjes integral $\int_a^b f(x)\, d\alpha(x)$. [2]

b. Evaluate the Stieltjes integral $\int_0^1 x\, dx^3$ using the limit definition. [5]

3. a. State the existence condition for a double integral $\iint_R f(x,y)\, dA$. [1]

b. Describe the region $R_x = R[-1, 1, -\sqrt{1-x^2}, \sqrt{1-x^2}]$. [2]

c. Change the Cartesian integral

$$\iint_{R_x} \frac{2}{(1+x^2+y^2)^2}\, dy\, dx$$

into an equivalent polar integral, and then evaluate this polar integral, where R_x is the region described in (b). [4]

SECTION "D"
[6Q. × 4 = 24 marks]

4. Use $\epsilon - \delta$ definition to verify that $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2} = 0$.

OR

If $f(x,y) = \begin{cases} \frac{x^3-y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$. Show that

a. $f(x,y)$ is continuous at $(0, 0)$

b. $f_1(0,0) = 1$ and $f_2(0,0) = -1$

5. Show that the differential forms in the integral

$$\int_{(0,0,0)}^{(0,1,1)} \sin y \cos x \, dx + \cos y \sin x \, dy + dz$$

is exact, and then evaluate the integral.

6. Prove that $f(x,y,z) = (x+y+z)^3 - 3(x+y+z) - 24xyz$ has a minimum at $(1,1,1)$.

7. Derive the relationship between the rectangular coordinates (x,y,z) and the spherical coordinates (ρ,ϕ,θ).

8. Prove that $\Gamma(n) = \frac{1}{n}\int_0^1 e^{-x^{1/n}} \, dx,\ n > 0$, and then show that $\int_0^\infty e^{-x^2} \, dx = \frac{\sqrt{\pi}}{2}$.

9. Evaluate the Stieltjes integral $\int_{-1}^{2} x^5 d\left(|x|^3\right)$ using integration by parts.

SECTION "D"

[5Q. × 2 = 10 marks]

10. Evaluate $\int_1^e \int_1^e \int_1^e \frac{1}{xyz} \, dx \, dy \, dz$.

11. Integrate $f(x,y) = x + y$ over the curve $C: x^2 + y^2 = 4$ in the counterclockwise sense.

12. Verify the Lagrange mean value theorem for the function $f(x) = x(x-1),\ 0 \le x \le \frac{1}{2}$.

13. Find the Fourier coefficient a_n for the function $f(x) = x,\ -\pi \le x \le \pi$ where the symbol has its usual meaning.

14. Compute $\frac{\partial u}{\partial x}$ for the system of equations $v + \ln u = xy, \quad u + \ln v = x - y$.