Marks scored:

KATHMANDU UNIVERSITY
End Semester Examination [C]
July, 2017

Level : B. Sc.		Course : MATH 201
Year : II		Semester : I
Exam Roll No. :	Time: 30 mins.	F. M. : 20
Registration No.:		Date : JUL 10 2017

SECTION "A"
[10Q. × 1 = 10 marks]

Fill in the blank space(s) by the most appropriate answer(s):

1. If $\vec{F}$ is independent of path within domain D enclosed by curve C, then $\int_C \vec{F}\cdot d\vec{r} =$ ______________________.

2. The function $f(x,y) = \dfrac{\sin(xy)}{x-y}$ is not continuous at points in a plane which lie along a line ______________________.

3. The function $f(x) = 1 - x^6$ has relative maximum at x = ________________.

4. If f(x) is an odd function, then the Fourier coefficient b_k = ____________ in the Fourier series $f(x) = \dfrac{a_0}{2} + \sum_{k=1}^{\infty}\left(a_k \cos kx + b_k \sin kx\right)$.

5. The cylindrical coordinates $(0, \theta, z)$ describes the ____________ axis.

6. $\int_0^2 x\, de^x =$ ______________________.

7. The Laplace inverse of $F(s) = \dfrac{1}{s(s-1)}$ is ______________________________.

8. The homogeneous function $f(x,y) = \sqrt{x} - \sqrt{y}$ is of degree ________________.

9. If $v + \log u = xy,\ u + \log v = x - y$ then $\dfrac{\partial u}{\partial x} =$ ________________________.

10. $\lim\limits_{(x,y,z)\to(1,1,1)} \dfrac{2xy + yz}{x^2 + z^2} =$ __________________________

SECTION "B"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s), DO NOT TICK, by choosing the most appropriate answers from among the given ones.

11. Let R be a region enclosed by a simple closed curve C in the plane oriented in counterclockwise. Then the area of the region R is given by the integral

____

[$\oint_C xdy - ydx$; $\oint_C ydx - xdy$; $\frac{1}{2}\oint_C xdy - ydx$; $\frac{1}{2}\oint_C ydx - xdy$]

12. The spherical coordinates for $x^2 + y^2 + (z-1)^2 = 1$ is ρ = ______________________.
[sinϕ; cosϕ; 2sinϕ; 2cosϕ]

13. The triple integral $\int_0^{2\pi}\int_{-\pi}^{\pi}\int_0^{a}\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta =$ ______________________.
[$4\pi a^2$; $4\pi a^3$; $\frac{4}{3}\pi a^2$; $\frac{4}{3}\pi a^3$]

14. The spherical coordinates of Cartesian coordinates (1, 0, 0) are ______________.
[(1, 0, 0); (1, 0, 1); (π/2, 1, 0); (1, π/2, 0)]

15. The function $f(x,y) = \dfrac{x^2 + y^2}{x^2 - 3x + 2}$ is not continuous at the points which lie along the line ______________________.
[x = -1; x = -2; x = 2; y = x]

16. A function $f(x, y)$ has an absolute maximum at a point (α, β) of a region R if and only if ______________________ for all (x, y) in R.
[$f(\alpha,\beta) > f(x,y)$; $f(\alpha,\beta) \geq f(x,y)$; $f(\alpha,\beta) < f(x,y)$; $f(\alpha,\beta) \leq f(x,y)$]

17. If a vector field $\vec{F}(x, y, z)$ is conservative, then $\vec{F} =$ ______________ for some scalar potential function *f*.
[∇f; $\nabla^2 f$; $\nabla \cdot (\nabla f)$; $\nabla \cdot (\nabla^2 f)$]

18. $f \in C^1 \Rightarrow f$ is ______________________.
[not differentiable; differentiable; continuous but not differentiable; continuous only]

19. The Stieltjes integral $\int_a^b f(x) d\alpha(x)$ exists if ______________________, $a \leq x \leq b$.
[$f(x) \in C$; $\alpha(x) \in \uparrow$; $\alpha(x) \in \downarrow$; $f(x) \in C$ and $\alpha(x) \in \uparrow$]

20. $\lim_{x \to 0+} e^{-1/x} =$ ______________________.
[0; 1; +∞; -∞]

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Level	: B. Sc.	Course	: MATH 201
Year	: II	Semester	: I
Time	: 2 hrs. 30 mins.	F. M.	: 55

SECTION "C"

[3 Q. × 7 = 21 marks]

1. Let $f(x,y) \in C^1$, (x,y) in region R and $f(x,y)$ is a homogeneous function of degree n in R, then prove that $x f_1(x,y) + y f_2(x,y) = n f(x,y)$, (x,y) in R. Verify this result for the function $f(x,y) = x^2 + y^2$. [4 + 3]

2. State and prove the convolution theorem of Laplace transform, and use this theorem to evaluate the convolution of the functions $f(t) = t$ and $g(t) = e^t$. [4 + 3]

OR

State and prove the linearity property of the Laplace transform. Use this linearity property to the relation $e^{iat} = \cos at + i \sin at$ to evaluate the Laplace transform of the functions $\cos at$ and $\sin at$. [3 + 4]

3. State Fubini's triple integral theorem for a rectangular parallelopiped region. Evaluate $\iiint_S \sqrt{x^2 + y^2}\, dx\, dy\, dz$, where S is the solid bounded by the surface $x^2 + y^2 = z^2$, $z = 0$, $z = 1$. [2 + 5]

SECTION "D"

[6 Q. × 4 = 24 marks]

4. Evaluate $\int_0^1 x^2 dx^2$ using definition of Stieltjes integral.

5. Change the Cartesian integral $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2)\, dx\, dy$ into equivalent polar integral and then evaluate the polar integral.

6. State a conservative vector field. If a vector field $\vec{F}(x,y) = M(x,y)\,\vec{i} + N(x,y)\,\vec{j}$ is conservative, then show that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

OR

Does the vector field $\vec{F}(x,y,z) = yz\,\vec{i} + xz\,\vec{j} + xy\,\vec{k}$ conservative? If yes, find the potential function for it.

7. Find the Fourier series of $f(x) = x, -\pi \le x \le \pi$.

8. Find a Cartesian coordinates equation of the sphere $\rho = 2\cos\phi$.

9. If $f(x,y) = \sqrt{|xy|}$, show that $f(x,y)$ is not differentiable at (0, 0).

<u>SECTION "E"</u>

[5 Q. × 2 = 10 marks]

10. Use the Laplace transform to solve $y'(t) + a\,y(t) = 1, y(0) = 0$, where a is a constant.

11. If $u = x^y$, $x = r\cos\theta$, $y = r\sin\theta$, find $\frac{\partial u}{\partial r}$.

12. Evaluate $\int_1^2 \int_y^{y^2} dx dy$.

13. Show that $\int_0^4 x d[x] = 10$, where $[x]$ is the greatest integer function.

14. Find the spherical equation of the Cartesian equation $x^2 + y^2 + \left(z - \frac{1}{2}\right)^2 = \frac{1}{4}$.