KATHMANDU UNIVERSITY
End Semester Examination
February/March, 2018

Marks Scored:

Level : B.Sc.
Year : II

Course : MATH 201
Semester: I

Exam. Roll No:

Time: 30 mins

F.M : 20

Registration No.:

Date : MAR 04 2018

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by the most appropriate answer(s):

1. The Cylindrical equation of the Cartesian equation $x^2 + y^2 + z^2 = 2$ is ____________________.

2. $\lim_{x \to 0-} e^{-1/x} =$ ____________________.

3. $\int_0^2 x\, dx^2 =$ ____________________.

4. If $\vec{F}$ is a conservative vector field on a domain D enclosed by a boundary curve C, then $\int_C \vec{F} \cdot d\vec{r} =$ ____________________.

5. The function $f(x) = 1 - x^6$ has a relative maximum at $x =$ ____________________.

6. If $x = r\cos\theta$ and $y = r\sin\theta$, then the Jacobian $\frac{\partial(x,y)}{\partial(r,\theta)} =$ ____________________.

7. The Laplace inverse of $\frac{1}{(s+a)(s+b)}$ is ____________________.

8. $\int_0^{\pi} \int_0^{\sin x} y\, dy\, dx =$ ____________________.

9. If $f(x)$ is an even function, then the Fourier coefficient $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin x\, dx$ is ____________________.

10. If $u = x^y$, then $\frac{\partial u}{\partial x} =$ ____________________.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), DO NOT TICK, by choosing the most appropriate answers from among the given ones.

11. If (x, y, z) and (ρ, ϕ, θ) be respectively the Cartesian and Spherical coordinates of a point in space then $z =$ ______________________

[$\rho \sin\phi$; $\rho \sin\theta$; $\rho \cos\phi$; $\rho \cos\theta$]

12. A function of m independent variables will have ______________________ derivatives of order n.

[n^m; m^n; n^{2m}; m^{2n}]

13. Let $f(x) \in C$ in $a \le x \le b$, $\alpha(a) = \lambda$, $\alpha(x) = \lambda + h$ in $a < x \le b$. Then $\int_a^b f(x)\, d\alpha(x) =$

______________.

[h; $f(a)$; $\alpha(a)h$; $f(a)h$]

14. The mass of a thin wire described by the curve $C: x^2 + y^2 = 1$ with density $\delta = 2$ is

____

[π; 2π; 3π; 4π]

15. Suppose $f(x, y) \in C^2$, $f_1 = f_2 = 0$ at (a, b) and $f_{11}f_{22} - f_{12}^2 < 0$ at (a, b), then $f(x, y)$ has a ______________________ at (a, b).

[relative maximum; relative minimum; saddle point; point of inflection]

16. If $f(x, y) = (x + y)^3$, then $\frac{\partial^3 f}{\partial \xi_\alpha^3} =$ ______________________, where the symbol has its usual meaning.

[$(\cos\alpha + \sin\alpha)^2$; $2(\cos\alpha + \sin\alpha)^2$; $(\cos\alpha + \sin\alpha)^3$; $6(\cos\alpha + \sin\alpha)^3$]

17. The first shifting theorem states that if $L\{f(t)\} = F(s)$, then $L\{e^{at} f(t)\} =$

______________________.

[$F(s+a)$; $-F(s+a)$; $-F(s-a)$; $F(s-a)$]

18. $\lim\limits_{h \to 0} \frac{f(x+2h) - 2f(x+h) + f(x)}{h^2} =$ ______________________.

[$f''(x)$; $\frac{f''(x)}{2}$; $-f''(x)$; $-\frac{f''(x)}{2}$]

19. The Cartesian integral $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy\, dx$ is equivalent to the polar integral

______________________.

[$\int_0^{2\pi} \int_{-1}^{1} dr\, d\theta$; $\int_0^{2\pi} \int_{-1}^{1} r dr\, d\theta$; $\int_0^{2\pi} \int_0^1 dr\, d\theta$; $\int_0^{2\pi} \int_0^1 r dr\, d\theta$]

20. The function $f(x, y) = |x|(1 + y)$ is ______________________ at $(0,0)$.

[not continuous; continuous; differentiable; differentiable but not continuous]

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MAR 0 4 2018

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Year : II
Time : 2 hrs. 30 mins.

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Semester: I
F.M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. State and prove Rolle's theorem. Verify Rolle's theorem for the function $f(x)=x^3(1-x)^5, \quad 0\le x\le 1$. [1+4+2]

OR

If $f(x,y)$ is a homogeneous function of degree n in a region R, and $f(x,y)\in C^1$ for all $(x,y)\in R$, then prove that

$$x f_1(x,y)+y f_2(x,y)=n f(x,y)$$

This is Euler's theorem. Verify the Euler's theorem for a function $f(x,y)=x^{1/3}y^{-4/3}$. [4+3]

2. Let D be a simply connected domain with a positively oriented piecewise smooth boundary curve C. If the vector field $\vec{F}(x,y)=M(x,y)\vec{i}+N(x,y)\vec{j}$ is continuously differentiable on D, then prove that

$$\oint_C M(x,y)dx+N(x,y)dy=\iint_D\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dA$$

where D is a standard domain, that is, no vertical or horizontal lines intersects the boundary more than twice. Verify this result for a vector field $\vec{F}(x,y)=x\vec{i}+y\vec{j}$ over the domain of integration $D: x^2+y^2\le a^2$.

3. Define Stieltjes integral $\int_a^b f(x)d\alpha(x)$ and use it to evaluate the integral $\int_0^2 f(x)d\alpha(x)$, where $f(x)=3$ and $\alpha(x)=\begin{cases}0, & 0\le x\le 1\\ 1, & 1<x\le 2\end{cases}$.

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Show that the function $f(x,y,z)=2xyz-4zx-2yz+x^2+y^2+z^2-2x-4y+4z$ has a minimum at $(1,2,0)$.

5. Evaluate $\lim_{x\to 0}\left(\frac{1}{x}\right)^{\tan x}$.

6. Use Convolution theorem to find the Laplace inverse of $\frac{1}{s(s+1)}$.

OR

Use Laplace transform method to solve the differential equation $y'(t)+ay(t)=1+e^t$, general solution.

7. Evaluate $\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0}\frac{2}{1+\sqrt{x^2+y^2}}\,dy\,dx$.

8. If $u=2xy, v=x^2-y^2, x=r\cos\theta$ and $y=r\sin\theta$. Eliminate x and y, and thus compute the Jacobian $\frac{\partial(u,v)}{\partial(r,\theta)}$.

9. Find the Fourier series of $f(x)=x, \ -\pi\le x\le\pi$.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Find the Spherical equation of the Cartesian equation $x^2+y^2+(z-3)^2=9$.

11. Evaluate the Stieltjes integral $\int_{0}^{2} x^2 d\,x^2$.

12. If $f(x,y)=x\tan^{-1}(x^2+y)$, find $f_2(1,0)$.

13. Evaluate the Triple integral $\int_0^a\int_0^b\int_0^c dx\,dy\,dz$.

14. Find the work done by a force $\vec{F}(x,y,z)=xy\,\vec{i}+y\,\vec{j}-yz\,\vec{k}$ over the curve $\vec{r}(t)=t\,\vec{i}+t^2\vec{j}+t\,\vec{k}, \quad 0\le t\le 1$.