KATHMANDU UNIVERSITY
End Semester Examination
February/March, 2019

Marks scored:

Level : B.Sc.
Year : II

Course : MATH 201
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date FEB 21 2019

SECTION "A"

[10 Q. × 1= 10 marks]

Fill in the blank space(s) by the most appropriate answer(s):

1. The equation in Cartesian system of the equation in spherical system $\rho = 3$ is ________________.

2. The value of the Triple integral $\int_0^a \int_0^a \int_0^a dx\, dy\, dz =$ ______________________.

3. The line integral $\int_C ds =$ _______________________, where C is a path along a line $y = 2x$ from $x = 0$ to $x = 1$.

4. The Laplace transform of the function $f(t) = t + e^t,\ t \geq 0$ is _______________________.

5. $\lim\limits_{x \to 0-} \frac{1}{1+e^{1/x}} =$ _______________.

6. If $\alpha(x) \in \uparrow, a \leq x \leq b$ and $f(x)$ is a _______________________ function on $[a, b]$, then the Stieltjes integral $\int_a^b f(x)\, d\alpha(x)$ exists.

7. The Fourier coefficient $a_0$in the Fouries series $f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ of the function $f(x)$ is given by $a_0 =$ ______________________________.

8. The directional derivative of the function $f(x, y) = x^2 + y^2$ at the point (1,1) in the direction of unit vector $\vec{i}$ is _____________________.

9. The double integral $\iint_R dA$ represents the ______________________ of the region R.

10. If $u = x^y$, then $\frac{\partial u}{\partial x} =$ ___________________________.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), DO NOT TICK, by choosing the most appropriate answers from among the given ones.

11. The graph of the function $f(x) = 1 + x + x^5$ has a point of inflection at $x =$ __________________.
[$-\frac{1}{5}$; 0; $\frac{1}{5}$; $\pm\frac{1}{\sqrt{5}}$]

12. The function $f(x, y) = \frac{x^2+y^2}{x-y}$ has point(s) of discontinuities along the curve __________________.
[$y = x$; $y = -x$; $y^2 = x^2$; $y^2 = -x^2$]

13. A definite integral of the form $\int_a^b f(x)d\alpha(x)$ is said to be a(n) ______________________ integral.
[Riemann; Cauchy; Stieltjes; Euler]

14. Let $L\{f(t)\} = F(s)$, then $L\{e^{at}f(t)\} = F(s-a)$ is said to be a ______________________ theorem.
[First shifting; Second Shifting; Laplace; Convolution]

15. Suppose $\delta = 1$ be the density of a planner lamina bounded by the lines $x + y = 1,\ x = 0$ and $y = 0$. Then the mass of the lamina is ______________________ unit.
[$\frac{1}{3}$; $\frac{1}{2}$; 1; $\frac{3}{2}$]

16. The function $f(x,y) = x^{\frac{1}{3}}\, y^{-\frac{4}{3}} \sin\left(\frac{y}{x}\right)$ is a homogeneous function of degree ______________________.
[0; 1; $\frac{1}{2}$; -1]

17. If a vector field $\vec{F}$ is conservative on a simply connected domain D enclosed by a curve C, then $\oint_C \vec{F} \cdot d\vec{r} =$ ______________________.
[-1; 0; 1; ∞]

18. If $x = r\cos\theta$ and $y = r\sin\theta$, then the Jacobian $\frac{\partial(x,y)}{\partial(r,\theta)} =$ ______________________.
[θ; r ; x ; y]

19. The value of the integral $\int_0^1 \int_0^y 2\, e^{y^2} dx\, dy =$ ______________________.
[e; $1-e$; $e-1$; $\frac{1}{e}$]

20. The spherical coordinates for $x^2 + y^2 + (z-1)^2 = 1$ is ______________________.
[$\sin\phi$; $\cos\phi$; $2\sin\phi$; $2\cos\phi$]

KATHMANDU UNIVERSITY
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FEB 2 1 2019

Level : B.Sc.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 201
Semester : I
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. State and prove Rolle's theorem for a function $f(x),\ a \le x \le b$. Verify the Rolle's theorem for a function $f(x) = x^2 - x,\ 0 \le x \le 1$. [2 + 3 + 2]

OR

Prove that the transformation $u = f(x,y),\ v = g(x,y)$ with Jacobian $J = \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} \neq 0$ have an inverse with jacobian $j = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$. For the function $u = x + y$ and $v = x - y$, verify that $J\,j = 1$. [5 + 2]

2. (a) Let C be a smooth curve parameterized by $\vec{r}(t),\ a \le t \le b$, and $\vec{F}$ be a continuous force vector over a region containing C. Then, define the work done by the force $\vec{F}$ along C. Write different ways to write work integral for $\vec{F}(x,y,z) = M(x,y,z)\vec{i} + N(x,y,z)\vec{j} + P(x,y,z)\vec{k}$ over the curve $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k},\ a \le t \le b$. [1 + 2]

(b) Find the work done by the force field $\vec{F} = (y - x^2)\,\vec{i} + (z - y^2)\vec{j} + (x - z^2)\,\vec{k}$ along the curve C : $\vec{r}\,(t) = t\,\vec{i} + t^2\,\vec{j} + t^3\vec{k}, 0 \le t \le 1$. [4]

3. State Fubini's stronger form for double integral of $f(x,y)$ over a non-rectangular region R. Change the Cartesian integral

$$\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{0} \frac{4\sqrt{x^2+y^2}}{1+x^2+y^2}\, dx\, dy$$

into equivalent polar integral, and then evaluate the integral. [2 + 5]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Evaluate the Stieltjes integral $\int_0^2 x\, dx^2$ using definition.

5. Use L'Hopital rule to evaluate the following limits: (a) $\lim_{x\to 0} x \ln x$ (b) $\lim_{x\to \pi/2} (\tan x)^{\cos x}$.

6. State linearity property of Laplace transform. Use this property to find the Laplace transform of the function $\sin at$, where a is a constant.

OR

Find the Fourier series of $f(x) = x, \quad -\pi \le x \le \pi$.

7. Find the integral of $f(x,y,z) = \frac{\sqrt{3}}{x^2+y^2+z^2}$ over the curve $\vec{r}\,(t) = t\,\vec{\imath} + t\,\vec{\jmath} + t\,\vec{k}\,,\ 1 \le t < \infty$.

8. Find the relative maximum, relative minimum and saddle point of the function $f(x,y) = x^2 - xy + y^4$.

9. If r and θ are polar coordinates, show that the directional derivative of f in the direction of θ, that is, $\left.\frac{\partial f}{\partial \xi_\theta}\right|_{(r,\theta)} = f_1(r,\theta)$, where the symbols have their usual meanings.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Find the equation and inequality of $\rho = 1,\ 0 \le \phi \le \frac{\pi}{2}$ in Cartesian system.

11. Show that the vector field $\vec{F}\,(x,y) = (x-y)\,\vec{\imath} + (y-x)\,\vec{\jmath}$ is conservative.

12. Evaluate the double integral $\int_0^1 \int_1^2 x\,y\,e^x\,dy\,dx$.

13. Compute $\frac{\partial u}{\partial x}$ for the following system of equations:

$$v + \ln v = xy$$
$$u + \ln v = x - y$$

14. Evaluate the Stieltjes integral $\int_0^1 x^2 dx^2$ using Stieltjes integral as a Riemann integral.