KATHMANDU UNIVERSITY
End Semester Examination [C]
December, 2024

Marks Scored:

Level : B.Sc.
Year : II
Course : MATH 201
Semester : I

Exam Roll No. :
Time: 30 mins.
F. M. : 20

Registration No.:
Date 23 DEC 2024

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by most appropriate words or symbol(s):

1. The function $f(x,y) = x^{\frac{1}{3}} y^{-\frac{4}{3}} \cos^{-1}\left(\frac{x^2}{y^2}\right)$ is a homogeneous function of degree _____.

2. The work done by a force field $\vec{F}(x,y,z) = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$ over the curve $C: \vec{r}(t) = \vec{i} + t\,\vec{j} + \vec{k},\ 0 \le t \le 1$ is _____.

3. The value of $\alpha$ in the mean value theorem, where $0 < \alpha < 2$ and $f(x) = x^2$ is _____

4. $\lim_{x \to 0+} e^{-1/x^2} =$ _____.

5. The function $f(x) = 1 + x^4$ has a relative minimum at $x =$ _____.

6. $\int_0^1 x^2\, d\,x^2 =$ _____.

7. If $f(x) = x^4,\ -2 \le x \le 2$ with a period 4. Then the value of the Fourier coefficient $b_n$ in the Fourier series $f(x) = a_0 + \sum_0^\infty a_n \cos nx + b_n \sin nx$ is _____.

8. The value of the integral $\int_0^a \int_0^b \int_0^c dx\, dy\, dz =$ _____.

9. The Cartesian coordinates $(x,y,z)$ of the cylindrical coordinates $(\sqrt{2}, 0, 1)$ is _____.

10. Assume that $\iint_R f(x,y)\, dA$ exists, then $\iint_R f(x,y) dA \ge 0$ if _____.

SECTION "B"

[10 Q. × 1 = 10 marks]

*Fill in the blank space(s),* ***DO NOT TICK,*** *by selecting the most appropriate answers from among the given ones.*

11. If $u + \ln u = xy$. Then $\frac{\partial u}{\partial x} =$ _____.

[ $\frac{ux}{u-1}$; $\frac{ux}{1+u}$; $\frac{uy}{u-1}$; $\frac{uy}{u+1}$ ]

12. The statement _____ _____is true if $f(x) = x,\ x \geq 0$ and $f(x) = 0, x < 0, -1 < x < 1$.

[ $f(x) \in C, f(x) \in C^1$; $f(x) \notin C, f(x) \notin C^1$;
$f(x) \notin C, f(x) \in C^1$; $f(x) \in C, f(x) \notin C^1$ ]

13. The function $f(x,y) = |x|(1+y)$ is _____ at (0,0)

[ differentiable; continuous;
both continuous and differentiable; continuous but not differentiable ]

14. The polar equation for the circle $x^2 + (y-3)^2 = 9$ is $r =$ _____.

[ $3\sin\theta$; $6\sin\theta$; $3\cos\theta$; $6\cos\theta$ ]

15. If $\alpha(x)$ is a non-decreasing function in $a \leq x \leq b$, and $k$ is a constant, then $\int_a^b d\,(\alpha(x) + k) =$ _____.

[ $b - a$; $a - b$; $\alpha(b) - \alpha(a)$; $\alpha(a) - \alpha(b)$ ]

16. The value of $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) =$ _____.

[ $\sqrt{2}$; $\pi\sqrt{2}$; $\pi$; $\pi/\sqrt{2}$ ]

17. Consider an object described by the curve C with density $\rho(x,y)$, then the mass of the object is given by _____.

[ $\int_C ds$ ; $\int_C \rho(x,y)dx$ ; $\int_C \rho(x,y)dy$; $\int_C \rho(x,y)\,ds$ ]

18. The elementary volume $dV$ in Cylindrical coordinates system is _____.

[ $dr\,d\theta\,dz$; $\frac{1}{2}\,dr\,d\theta\,dz$; $r\,dr\,d\theta\,dz$; $r^2 dr\,d\theta\,dz$ ]

19. The equation of the cone $z^2 = x^2 + y^2$ in Spherical coordinates is _____.

[ $\phi = \frac{\pi}{2}$; $\phi = \frac{\pi}{4}$; $\theta = \frac{\pi}{2}$; $\theta = \frac{\pi}{4}$ ]

20. $\int_0^\infty e^{-x^2} dx =$ _____.

[ $\sqrt{\pi}$; $\sqrt{\frac{\pi}{2}}$; $\frac{\sqrt{\pi}}{2}$; $\frac{\pi}{2}$ ]

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23 DEC 2024

Course : MATH 201
Semester : I
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1.

a. State and prove Rolle's theorem for a function $f(x)$. [1+3]
b. If $f(x) = x^2$, prove that Rolle's theorem is applicable in $-1 \leq x \leq 1$, and verify the theorem. [3]

**OR**

a. Let R be a region, $f(x,y,z) \in C^1$, $(x,y,z) \in R$ and $f(x,y,z)$ is a homogeneous function of degree $n$ in R. Then prove that $x f_1(x,y,z) + y f_2(x,y,z) + z f(x,y,z) = n f(x,y,z)$. [4]
b. If $u = x^2y + y^2z + z^2x$, then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = (x+y+z)^2$. [3]

2.

a. Define the Fourier series. [1]
b. Find the Fourier series of $f(x) = x^2$, $-\pi \leq x \leq \pi$. [3]
c. Prove that the Beta function $B(m,n) = 2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$. [3]

3.

a. Use the limit definition of double integral to evaluate the integral $\int_0^1 \int_0^1 x \, dx \, dy$ [4]
b. Find a center of mass of a thin plate of density $\delta = 3$ bounded by the lines $x = 0$, $y = x$, and the parabola $y = 2 - x^2$ in the first quadrant. [3]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Use the $\epsilon - \delta$ definition to verify that $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2} = 0$.

**OR**

Discuss the continuity of $f(x,y) = \begin{cases} \frac{x^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = 0 \end{cases}$ at (0, 0).

5. Find the work done by $\vec{F}(x,y,z) = xy\,\vec{i} + y\,\vec{j} - yz\,\vec{k}$ over the curve $\vec{r}(t) = t\,\vec{i} + t^2\,\vec{j} + t\,\vec{k}$, $0 \leq t \leq 1$.

6. Show that the function $f(x,y,z) = (x+y+z)^3 - 3(x+y+z) - 24\,xyz + 124$ has a maximum at $(-1,-1,-1)$.

**P.T.O.**

7. Evaluate the Stieltjes integral $\int_{-1}^{2} x^5 \, d(|x|^3)$.

8. If $(x, y, z)$ and $(\rho, \phi, \theta)$ represent a point in space in Cartesian and Spherical coordinates systems, respectively. Then derive the relation for $x, y,$ and $z$ in terms of $\rho, \phi,$ and $\theta$.

9. If $f(x,y) = \begin{cases} \frac{x^3+2y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$. Show that $f_1(0,0)$ and $f_2(0,0)$ exist but $f(x,y)$ is not differentiable at $(0,0)$.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Describe the region $R_x = R[0, 2, -x, x]$.

11. Find the parametric equation of $y = 2x^2, \ 0 \leq x \leq 2$.

12. Convert to polar coordinates and evaluate the integral $\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 2x \, dy \, dx$.

13. Graph the sets of points whose polar coordinates satisfy the conditions $1 \leq r \leq 2$, and $0 \leq \theta \leq \frac{\pi}{2}$.

14. Write the Cartesian equation $x^2 + y^2 + (z-3)^2 = 9$ in the Spherical coordinates system.