

KATHMANDU UNIVERSITY
End Semester Examination [C]
November/December, 2023

Marks Scored:

Level : B.Sc.

Year : II

Exam Roll No.:

Time: 30 mins.

Course : MATH 201

Semester : I

F. M. : 20

Registration No.:

Date **29 NOV 2023**

SECTION "A"
[10Q. × 1 = 10 marks]

Fill in the blank space(s) by the most appropriate word(s) or symbol(s).

1. If n is a positive integer, then $\Gamma(n + 1) =$ _____.
2. If $u + \ln u = xy$, then $\frac{\partial u}{\partial x} =$ _____.
3. The center of the circle $r = 2 \cos \theta$ is _____.
4. The cylindrical coordinates (r, θ, z) of the Cartesian coordinates $(1, 1, 1)$ is _____.
5. $\iint_R dA =$ _____, where $R: x^2 + y^2 \leq a^2$.
6. $\lim_{x \rightarrow 0^-} e^{-1/|x|} =$ _____.
7. The Stieltjes integral $\int_{-1}^1 x d|x| =$ _____.
8. Let C be the boundary of a region R , and $\int_C \vec{F} \cdot d\vec{r} = 0$ around any closed loop C in R , then the field \vec{F} is _____ on R .
9. The mass of an object described by the curve C with density $\delta(x, y, z)$ is given by _____.
10. $\lim_{(x,y) \rightarrow (0,0)} \frac{x-y+2\sqrt{x}-2\sqrt{y}}{\sqrt{x}-\sqrt{y}} =$ _____ when $x \neq y$

SECTION "B"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s), DO NOT TICK, by choosing the most appropriate answers from among the given ones.

11. $B(m, n) =$ _____, $m > 0, n > 0$ where the symbols have their usual meanings.
 $\left[\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}, \quad \frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)}, \quad \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n+1)}, \quad \frac{\Gamma(m+n+1)}{\Gamma(m)\Gamma(n)} \right]$
12. The value of integral $\int_0^t \left(\ln \frac{1}{x}\right)^{p-1} dx$, $p > 0$ is _____.
 $\left[\Gamma(p); \quad p \Gamma(p); \quad \Gamma(p+1); \quad \frac{\Gamma(p)}{p} \right]$

13. The polar equation of the Cartesian equation $y^2 = x$ is _____.
 [$\cos \theta \sin \theta$; $\cot \theta \csc \theta$; $\tan \theta \sin \theta$; $\cot \theta \sin \theta$]
14. If (r, θ, z) and (ρ, ϕ, θ) are respectively representing the Cartesian coordinates and spherical coordinates, then _____.
 [$r = \rho \cos \phi$; $r = \rho \sin \theta$; $z = \rho \sin \phi$; $z = \rho \cos \phi$]
15. If $f(x) = |x|$, then $f'_+(0) =$ _____.
 [-1; 0; 1; ∞]
16. The directional derivative of $f(x, y)$ at a point (a, b) in the direction of unit vector \vec{i} is given by _____.
 [$\frac{\partial f}{\partial x}$; $\frac{\partial f}{\partial y}$; $\frac{\partial^2 f}{\partial x^2}$; $\frac{\partial^2 f}{\partial y^2}$]
17. If $u = f(x, y)$, $y = g(x, z)$, then $\frac{\partial u}{\partial y} =$ _____, when y and z are treated as independent variables.
 [$\frac{1}{g_1}$; $\frac{1}{f_1 g_1}$; $f_2 + \frac{f_1}{g_1}$; $f_1 + \frac{f_2}{g_1}$]
18. If $f(x) \in C$, $a \leq x \leq b$ and $\alpha(x) \in C^1$, $a \leq x \leq b$, then $\int_a^b f(x) d\alpha(x) =$ _____.
 [$\int_a^b f(x) \alpha(x) d\alpha$; $\int_a^b f(x) \alpha(x) d f(x)$;
 $\int_a^b f(x) \alpha'(x) dx$; $\int_a^b f'(x) \alpha(x) dx$]
19. Let R be the region enclosed by a simple closed curve Γ in the plane oriented counterclockwise. Then the area of the plane region R using Green's theorem in a plane is given by the integral _____.
 [$\frac{1}{2} \int_R x dy + y dx$; $\frac{1}{2} \int_R y dy - x dx$;
 $\frac{1}{2} \int_R (x + y) dy - y dx$; $\frac{1}{2} \int_R x dy - y dx$]
20. If $f(x, y)$ is a homogeneous function of degree n , then $x f_1(x, y) + y f_2(x, y) =$ _____.
 [$f(x, y)$; $n f(x, y)$; $n(n - 1)f(x, y)$; $xy f(x, y)$]

KATHMANDU UNIVERSITY
End Semester Examination [C]
November/December, 2023

29 NOV 2023

Level : B.Sc.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 201
Semester : I
F. M. : 55

SECTION "C"

[3Q. × 7 = 21 marks]

1. a. State and prove the tangential form of Green's theorem. [4]
b. Verify the tangential form of Green's theorem for the vector field $\vec{F}(x, y) = (x - y)\vec{i} + x\vec{j}$ and the region R bounded by the unit circle $C: \vec{r}(t) = (\cos t)\vec{i} + (\sin t)\vec{j}$, $0 \leq t \leq 2\pi$. [3]
2. a. Let D be a domain. If $f(x, y) \in C^1$ in D, and the circle $(x - a)^2 + (y - b)^2 \leq \delta^2$ lies in D, then prove that $f(a + \Delta x, b + \Delta y) - f(a, b) = f_1(a + \theta_1 \Delta x, b) \Delta x + f_2(a + \Delta x, b + \theta_2 \Delta y) \Delta y$ where $\Delta x^2 + \Delta y^2 < \delta^2$ and $0 < \theta_1 < 1$, $0 < \theta_2 < 1$. [4]
b. If $u = f(x, y)$, $x = g(r, s)$, $y = h(t) k(r)$ find $\frac{\partial u}{\partial r}$, $\frac{\partial u}{\partial s}$ and $\frac{\partial u}{\partial t}$. [3]

OR

- a. Let R be a region. If $f(x, y) \in C^1$ for all (x, y) in R and $f(x, y)$ is homogeneous of degree n in R, then prove that $f_1(x, y)x + f_2(x, y)y = n f(x, y)$ for all (x, y) in R. [4]
b. Verify the result in (a) for the function $f(x, y) = x \cos\left(\frac{y}{x}\right)$. [3]
3. a. Define the limit form of the Stieltjes integral, and state the condition for the existence of the Stieltjes integral. [2]
b. Evaluate the Stieltjes integral $\int_0^1 x dx^2$ using the limit definition of the Stieltjes integral. [5]

SECTION "D"

[6 Q. × 4 = 24 marks]

4. Evaluate $\int_0^{\sqrt{2}} \int_0^{3y} \int_{x^2+3y^2}^{8-x^2-y^2} dz dx dy$.
5. If $z(z^2 + 3x) + 3y = 0$, prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{2z(x-1)}{(z^2+x)^3}$.
6. Describe the region $R_y = R[-4, 5, -\sqrt{25 - y^2}, 3y/4]$. Express it as the sum of several regions R_x .
7. Find the extremum values of the function $f(x, y, z) = x^4 + x^2y + y^2 + z^2 + xz + 1$.
8. Use the property of the Beta and Gamma function to evaluate the integral $\int_0^2 x(8 - x^3)^{1/3} dx$.

- Evaluate the Fourier series corresponding to the function $f(x) = x^2$, $-\pi \leq x \leq \pi$ with period 2π .
9. If r, θ are polar coordinates, show that $\left. \frac{\partial f}{\partial \xi_\theta} \right|_{(r, \theta)} = f_1(r, \theta)$ where the symbols have their usual meanings.

SECTION "E"

[5 Q. \times 2 = 10 marks]

10. Evaluate the Stieltjes integral $\int_0^\pi \cos x \, d \sin x$.
11. Evaluate the double integral $\int_0^{\frac{\pi}{4}} \int_0^{\sec x} y^3 \, dy \, dx$.
12. Establish that $\lim_{(x,y) \rightarrow (0,0)} x^2 + y^2 = 0$.
13. Find the equivalent cylindrical equation and inequality of the Cartesian equation and inequality
$$z = \sqrt{x^2 + y^2}, 1 \leq z \leq 2.$$
14. Find f_1 if $f(x, y) = x^{xy}$.