KATHMANDU UNIVERSITY
End Semester Examination [C]
April/May, 2023

Marks Scored:

Level : B.Sc.
Year : II
Exam Roll No.:
Registration No.:

Time: 30 mins.

Course : MATH 201
Semester : I
F. M. : 20
Date : 27 APR 2023

SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) by the most appropriate answer(s).

1. If $u = f(g(t), h(t))$, then $\frac{du}{dt} =$ ____________________.

2. Suppose $f(x, y) \in C^1$, and $\frac{\partial f}{\partial \xi_\alpha}$ be the directional derivative of f in the direction ξ_α, then $\frac{\partial f}{\partial \xi_\alpha} =$ ____________________.

3. $\int_a^b [x]\, dx =$ ____________________, where $[x]$ is the greatest integer function.

4. $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) =$ ____________________.

5. The slope of the straight line represented by the polar equation $r = \frac{4}{2\cos\theta - \sin\theta}$ is ____________________.

6. The graph of the function $f(x) = x^5 + x + 1$ has a point of inflection at $x =$ ____________________.

7. $\int_0^a \int_0^b \int_0^c dV =$ ____________________, where $dV = dx\, dy\, dz$.

8. The vector equation of the parabola $y = x^2,\ 0 \le x \le 2$ is $\vec{r}(t) =$ ____________________, $0 \le t \le 2$.

9. $\frac{\partial^2}{\partial r^2}(r^2 + s) =$ ____________________.

10. Let $f(x, y) = x,\ R_x = R[0, 1, -x, x]$, then $\iint_{R_x} f(x, y)\, dA =$ ____________________.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answers from among the given ones.

11. If $f(x,y)$ is a homogeneous function of degree n, then $x\,f_1(x,y) + y\,f_2(x,y) =$ ________________.

    [$f(x,y)$, $n\,f(x,y)$, $n(n-1)f(x,y)$, $xy\,f(x,y)$]

12. $\lim\limits_{(x,y)\to(1,1)} \frac{x^2+xy-2y^2}{x-y} =$ ______________________.

    [-3, 0, 1, 3]

13. Let $f(x) \in C,\ a \le x \le b$ and $\alpha(x)$ is a step function with jumps h_k at the point c_k, where $a < c_1 < c_2 < \cdots < c_n < b$, then $\int_a^b f(x)\,d\alpha(x) =$ __________________________.

    [$h_n f(c_n) - h_1 f(c_1)$, $\sum_{i=1}^{n} h_i$, $\sum_{i=1}^{n} f(c_i)$, $\sum_{i=1}^{n} h_i\,f(c_i)$]

14. $f(x) = x\,(10-x), 0 < x < 10$, period = 10 is a(n) ______________________ function.

    [an odd, an even, not continuous, minimum at $x = 5$]

15. The Cartesian equation of the Cylindrical equation $r = -3\sec\theta$ is ________________.

    [$x = 3$, $x = -3$, $y = 3$, $y = -3$]

16. Let $D \subset \mathbb{R}^n$ and $f: D \to \mathbb{R}$. The point $A = (a_1, a_2, \cdots, a_n) \in D$ is said to be a ____________________ if $f(X) \le f(A)$ for all points $X = (x_1, x_2, \cdots, x_n)$ in the neighborhood of A.

    [local minimum, local maximum, global minimum, global maximum]

17. $\int_0^8 \int_{\sqrt[3]{x}}^{2} \frac{1}{1+y^4}\,dy\,dx =$ ________________________.

    [$\frac{17}{4}$, $\ln\frac{17}{4}$, $-\frac{17}{4}$, $\ln\frac{4}{17}$]

18. A vector field $\vec{F}(x,y) = M(x,y)\,\vec{i} + N(x,y)\,\vec{j}$ is conservative if ____________________.

    [$\frac{\partial N}{\partial y} = \frac{\partial M}{\partial x}$, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, $\frac{\partial^2 M}{\partial x \partial y} = \frac{\partial^2 N}{\partial x\,\partial y}$, $\frac{\partial^2 M}{\partial y^2} = \frac{\partial^2 N}{\partial x^2}$]

19. The radius of the circle $r = 4\cos\theta$ is ________________.

    [1, 2, 3, 4]

20. If $x = r\cos\theta,\ y = r\sin\theta$. Then $\frac{\partial(x,y)}{\partial(r,\theta)} =$ ________________.

    [0, $\sin\theta$, $\cos\theta$, r]

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End Semester Examination [C]
April/May, 2023

27 APR 2023

Level : B.Sc.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 201
Semester : I
F. M. : 55

SECTION "C"
[3Q. × 7 = 21 marks]

1. a. Define a homogeneous function for a function of two variables x and y. [1]

   b. State and prove Euler's theorem for a function of two variables x and y. [4]

   c. Verify Euler's theorem for a function $f(x, y) = 3x^2 + 5xy + y^2$. [2]

OR

   a. State and prove Lagrange Mean value theorem. [1 + 3]

   b. Use the Lagrange Mean value theorem to find the approximate value of $\sqrt{101}$. [3]

2. a. Suppose $f(x) \in \uparrow,\ \alpha(x) \in C,\ a \leq x \leq b$. Prove that [5]

$$\int_a^b f(x)\, d\alpha(x) + \int_a^b \alpha(x)\, df(x) = f(b)\, \alpha(b) - f(a)\, \alpha(a)$$

   b. Use the result in (a) to find the value of $\int_0^{\pi/2} x\, d \sin x$. [2]

3. a. Let $\vec{F}(x, y, z)$ be a conservative vector field on the simply connected domain D, and let $f(x, y, z)$ be a scalar potential function for $\vec{F}(x, y, z)$. Then if C is any piecewise smooth curve lying entirely within D with initial point A and terminal point B, then prove that [5]

$$\int_C \vec{F} \cdot d\,\vec{r} = f(B) - f(A)$$

   b. Find the work done by $\vec{F}(x, y, z) = xy\,\vec{i} + y\,\vec{j} - yz\,\vec{k}$ over the curve [2]

$$\vec{r}(t) = t\,\vec{i} + t^2\,\vec{j} + t\,\vec{k},\ \ 0 \leq t \leq 1.$$

SECTION "D"
[6 Q. × 4 = 24 marks]

4. If $z^3 + 3zx + 3y = 0$, prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{2z(x-1)}{(z^2+x)^3}$

5. Use the $\epsilon - \delta$ definition to verify $\lim_{(x,y)\to(0,0)} \frac{x^2 y^2}{x^2+y^2} = 0$

6. Find the equivalent polar integral of $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{1+x^2+y^2}\, dy\, dx$ and then evaluate the polar integral.

7. Verify Green's theorem for the line integral $\int_C -y\,dx + x\,dy$ where C: the line segment joining the points (-1, 0) and (1, 0) and the upper half portion of the circle $x^2 + y^2 = 1$.

OR

Compute $\int_C (x+y)dx$ if C: $\vec{r}(t) = \cos t\,\vec{\imath} + \sin t\,\vec{\jmath},\ 0 \le t \le \frac{\pi}{2}$.

8. Use the Gamma function property to evaluate the integral

$$\int_0^{\frac{\pi}{2}} \sin^6\theta\ \cos^4\theta\ d\theta$$

9. Find the extremum or point of inflection of the function

$$f(x,y) = x^3 + y^3 - 63(x+y) + 12\,xy.$$

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Find the Fourier series of $f(x) = x, \quad -\pi \le x \le \pi$.

11. If $f(x,y) = x^3 + xy,\ \alpha = \frac{\pi}{4}$. Find $\left.\frac{\partial f}{\partial \xi_\alpha}\right|_{(1.0)}$.

12. Evaluate the Stieltjes integral $\int_0^5 (x^2+1)dx^2$.

13. Find the Cartesian equation of the cylindrical equation $z + r^2\cos 2\theta = 0$.

14. If $f(x) = 1 + |x|$. Show that $f'(0)$ does not exist.