KATHMANDU UNIVERSITY
End Semester Examination [C]
May/June, 2019

Marks Scored:

Level : B.Sc./B. Pharm./B.Tech.
Year : I
Course : MATH 111
Semester: I

Exam Roll No. :
Time: 30 mins.
F.M. : 20

Registration No.:
Date 30 MAY 2019

SECTION "A"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The range of the function $f(x) = \sqrt{x}$ is _____.

2. If the graph of the line $y = x$ is shifted 1 unit downward and 1 unit left, then the equation of new line is _____.

3. If $\lim_{x\to 0} \frac{e^{ax}-1}{x} = 3$, then the value of a = _____.

4. The value of the $\lim_{x\to 1^-} f(x)$ = _____, if $f(x) = \begin{cases} x^2 - 2, & x \geq 1 \\ 1 + x, & x < 1 \end{cases}$.

5. The average rate of change of the function $f(x) = x^2 + 1$ on the interval $[-1, 1]$ is _____.

6. The critical point of the function $f(x) = 2x^2 - 4x + 1$ is x = _____.

7. The horizontal asymptote of the curve $y = \frac{4x^2-x+1}{2x^2-2}$ is _____.

8. $\int \frac{dx}{8+2x^2}$ = _____ + C.

9. The area bounded by the graph of the function $f(x) = e^x,\ 0 \leq x \leq 1$ with x-axis is _____.

10. The row rank of the matrix $A = \begin{pmatrix} 1 & -5 & 3 \\ 0 & 1 & -2 \\ 0 & -2 & 4 \end{pmatrix}$ is _____.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), DO NOT TICK, by selecting the most appropriate answers from among the given ones.

11. If $f(x) = x + 1$ and $g(x) = cosx$, then the value of the composite function $(fog)(x)$ at $x = \pi$ is _____
[−1; 0; 1; ∞]

12. The graph of a function $y = f(x)$ is symmetric about _____ if $f(-x) = -f(x)$
[x-axis; y-axis; line $y = x$; origin]

13. A function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = e^{-x^2}$ is a/an _____ function.
[odd; even;
neither odd nor even; discontinuous]

14. Function $f(x) = \begin{cases} x^2, & x \geq 0 \\ x - 1, & x < 0 \end{cases}$ has jump discontinuity at the point $x = 0$, then the height of the jump is _____ units.
[-1; 0; 1; ∞]

15. The function $f(x) = \frac{x-2}{x-1}$ has a _____ a/an discontinuity at the point $x = 1$.
[removable; jump;
infinite; oscillating]

16. The graph of the function $f(x) = x^2$ is decreasing on the interval _____.
[$(-\infty, \infty)$; $(-\infty, 0)$; $(0, \infty)$; $(-1,1)$]

17. If $F(x) = \int_0^{tanx} \frac{dt}{\sqrt{1+t^2}}$, then $\frac{dF}{dx} =$ _____.
[$sinx$; $cosecx$; $cosx$; $secx$]

18. If y varies directly as x, and $y = 2$ when $x = 0.5$, then _____.
[$y = \frac{1}{x}$; $y = 4x$; $y = x$; $y = \frac{x}{2}$]

19. A system of linear equations $AX = b$ is _____ if the row rank of the coefficient matrix A is not equal to the row rank of augmented matrix $(A; b)$_____.
[consistent; inconsistent; homogeneous; non-homogeneous]

20. The equation $x^2 - y^2 = 1$ graphically represents the ..... .
[circle; ellipse; parabola; hyperbola]

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30 MAY 2019

Level : B.Sc./B.Pharm./B.Tech.
Year : I
Time : 2 hrs. 30 mins.

Course : MATH 111
Semester : I
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define the derivative of a function $f(x)$ at a point $x = x_0$. Explain the geometrical meaning of derivative. When a bactericide was added to a nutrient broth in which bacteria were growing, the bacterium population continued to grow for a while, but then stopped growing and began to decline. If the size of the population at time t (in hours) was $P = 10^6 + 10^4 t - 10^3 t^2$, then calculate the following:
(a) The population at the time $t = 0$.
(b) The time at which the population doubles.
(c) The time at which population is maximum.

[1+2+0.5+1.5+2]

OR

State the conditions for a function $f(x)$ to attend extremum value at a point. For a given function $f(x) = 2x^3 + 3x^2 - 36x + 1$ defined on the closed interval $[-4, 4]$, calculate the following:
(a) Interval(s) on which the function of $f(x)$ increases and decreases.
(b) Find the absolute maximum and absolute minimum values of the function $f(x)$ on the given interval.

[2+2.5+2.5]

2. State the Fundamental Theorem of Integral Calculus to evaluate the value of the definite integral. Find the area of the propeller-shaped region enclosed by the curve $x - y^3 = 0$ and the line $x - y = 0$. [2+1+4]

3. Define consistent and inconsistent system of linear equations with examples. Show that the homogeneous system of linear equations

$$x_1 + 2x_2 + 2x_3 + 3x_4 = 0$$
$$2x_1 + 4x_2 + 3x_3 + 7x_4 = 0$$
$$x_1 + 2x_2 + x_3 + 4x_4 = 0$$

has infinitely many solutions and find the parametrically represented solution. Also, find any two non-zero specific solutions. [2+4+1]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Evaluate $\frac{dy}{dx}$ (*ANY TWO*):
(i) $x = 3(\theta - \sin\theta), y = 3(1 - \cos\theta)$

(ii) $y^2 = x^2 + \sin xy$

(iii) $y = \sqrt{4x^2 + 12x + 13}$

5. Evaluate the following integrals (*ANY TWO*):
   (i) $\int (x^2 + 3x + 5)e^{-x}dx$
   (ii) $\int \frac{dx}{\sqrt{4x^2+12x+13}}$
   (iii) $\int_{-1}^{1} x\sqrt{1-x^2}\,dx$

6. Find the limit $\lim_{x \to 1^+} (x-1)^{1-x}$.

OR

Find the asymptotes to the curve $y = \frac{x^2+1}{x^2-1}$ and use them to sketch the graph of the curve. **(Use Graph Paper).**

7. Find the equations of tangent and normal lines to the curve $y^2 - 2x - 4y - 1 = 0$ at the point $(-2,1)$.

8. Evaluate the definite integral $\int_0^1 (x^2 + 2x)dx$ by using the limit of a Riemann sum.

9. If y is the sum of two quantities of which one varies directly as x and the other inversely as x^2, and if $y = 6$ when $x = 4$ and $y = 15$ when $x = 8$, then find the relation between x and y. Also find the value of y when $x = 6$.

SECTION "E"
[5 Q. × 2 = 10 marks]

10. Find the solution set of the inequity $x^2 - x \leq 2$.

11. Evaluate the $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{sinx}\right)$.

12. Write the matrix $A = \begin{pmatrix} 1 & 1 & -2 & 1 \\ 0 & 1 & -1 & 3 \\ -1 & 4 & -3 & 14 \end{pmatrix}$ into its equivalent reduced echelon form and find the row rank of the matrix.

13. Define the definite integral of a function $f(x)$ on an interval $[a, b]$ as a limit of Riemann sum and interpret it geometrically.

14. Solve the differential equation $\frac{dy}{dx} = \frac{2x}{\sqrt{x^2+1}}$ with initial condition $y(0) = 2$.