KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

Marks Scored:

Level : B.Sc./B.Tech.
Year : I
Course : MATH 111
Semester : I

Exam Roll No. :
Time: 30 mins.
F. M. : 20

11 JUL 2023

Registration No.:
Date :

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbol(s).

1. A homogeneous system of equations has a solution which is known as _____ solutions.

2. If the function $y = \sqrt{x}$ is shifted 2 units right, then the resulting function is _____.

3. If a function x^2 grows faster than the function $\ln x$ as $x \to \infty$ then $\lim_{x\to\infty} \frac{x^2}{\ln x} =$ _____.

4. The derivative of $\tan^{-1}\frac{1}{x}$ is _____.

5. If the derivative of a function $f(x)$ is positive on (a, b), then $f(x)$ is _____ on (a, b).

6. $\frac{d}{dx}\int_2^{x^2} \cos t \; dt =$ _____.

7. A function $y = f(x)$ is continuous at a in $[a, b]$, if it is continuous from _____ at a.

8. The graph of a function $y = 2x^3$ is symmetrical about the _____.

9. A finite interval is said to be half open interval, if it contains one _____.

10. If C is a square matrix and $|C| \neq 0$, then the matrix C is called _____.

SECTION "B"
[10Q. × 1=10 marks]

Fill in the blank space(s), DO NOT TICK, by selecting the most appropriate answers from among the given ones.

11. A function $f(x)$ is symmetric about the _____ if $f(-x) = f(x)$.
[x-axis; y-axis; origin; the line $y = x$]

12. The domain of the function $f(x) = \sqrt{9 - x}$ is _____.
[$[9, \infty)$; [-3,3]; $(-\infty, 9]$; (-9, 9)]

13. The function $f(x) = \frac{x^2-4}{4-x}$ is discontinuous at $x =$ _____.
[$x = \pm 2$; $x = 2$; $x = 4$; $x = -4$]

14. The vertical asymptote of the curve $f(x) = \frac{x}{x+2}$ is _____.
[$x = 1$; $x = 0$; $x = 2$; $x = -2$]

15. A function has a critical point at $x = a$, if _____.
[$f(a) = 0$; $f'(a) = 0$; $f''(a) = 0$; $f'(a) \neq 0$]

16. The y-intercept of the line $2x + y = 5$ is _____.
[2; 5; 3; -5]

17. The average value of the function $f(x) = \frac{x^2}{2}$ on [0, 3] is _____.
[$-\frac{3}{2}$; $\frac{3}{2}$; $\frac{1}{2}$; $-\frac{1}{2}$]

18. If $f(x) = x^2$ and $g(x) = x + 1$, $(fog)(0)$ is _____.
[-1; 2; 1; 0]

19. $\lim_{x \to 1} \frac{x^2 - 1}{x+1} =$ _____
[1; 1/2; ∞; 0]

20. $\int \frac{3x}{x^2-3} dx =$ _____
[$\frac{1}{3}\ln(x^2 - 3)$; $\frac{3}{2}\ln(x^2 - 3)$; $(x^2 - 3)^2$; $\frac{1}{2}(x^2 - 3)$]

KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

11 JUL 2023

Level : B.Sc./B.Tech.
Year : I
Time : 2 hrs. 30 mins.

Course : MATH 111
Semester : I
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define a derivative of a function at a point. Show that if a function f has a derivative at a point $x = c$ then f is continuous at $x = c$. Show that the continuous function $f(x) = |x|$ is not differentiable at $x = 0$. [1 + 3 + 3]

OR

Define the continuity of a function at a point. Show that $g(x) = \frac{x^2-16}{x^2-3x-4}$ has a continuous extension to x = 4, and hence find the extension. For what value of b is

$$g(x) = \begin{matrix} x, & x < -2 \\ bx^2, & x \geq -2 \end{matrix}$$

continuous at every x ? [1+3+3]

2. Define local extreme values of a continuous function on an interval. Find the local maximum, minimum values and the point of inflection of $f(x) = x^3 - 3x + 3$. Use these information to sketch the graph. [1+4+2]

3. Define the consistent and inconsistent of a system of linear equations. Test the consistency of the following system of linear equations and solve it: [2+5]

$$y + 4z = 6$$
$$x + 3y + 5z = 2$$
$$3x + 7y + 7z = -6$$

SECTION "D"
[6Q. × 4 = 24 marks]

4. Find $\frac{dy}{dx}$ (*ANY TWO*).

a. $y = \left(\frac{x-1}{x+2}\right)^{-2}$ b. $y = \ln(\sin(\cos x))$ c. $y = x^{(1/x)}$

5. Evaluate (*ANY TWO*).

a. $\int \frac{x+5}{x^2-4x-5} dx$ b. $\int_1^2 x \ln x \, dx$ c. $\int \sqrt{1-\cos x} \, dx$

6. Define an ordinary differential equation. Solve the initial value problem.

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}, \quad y(0) = 0.$$

7. Find the length of the curve $y = x^{3/2}, \quad 0 \leq x \leq 4.$

8. Evaluate the following limits

a. $\lim_{x \to 0} \frac{\sin x - x}{x^3}$ (b) $\lim_{x \to \infty} (\ln x)^{\frac{1}{x}}$

9. Find the inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$, if it exist.

SECTION "E"

[5Q. × 2 = 10 marks]

10. A cube's surface area increases at the rate of $96\, cm^2/sec$. At what rate is the cube's volume changing when the edge length $x = 4cm$?

11. Find the row rank of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 7 & 8 \end{bmatrix}$.

12. Find the linearization of the function $f(x) = \ln(1 + x)$ at $x = 0$.

13. Find the vertical asymptotes of the curve $y = -\frac{8}{x^2 - 4}$.

14. Find the tangent to the curve $y = x^2 + 1$ at $(2, 5)$.