KATHMANDU UNIVERSITY
End Semester Examination [C]
November/December, 2023

Marks Scored:

Level : B.Sc./B.Tech.
Year : I

Course : MATH 111
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : 30 NOV 2023

SECTION "A"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbol(s).

1. A function $y = f(x)$ is shifted k units right, the resulting function is _____

2. The graph of the equation $y = ax^2 + bx + c, a \neq 0$ is a parabola which opens upward if _____.

3. The derivative of $y = \int_1^{x^2} \cot t \ dt$ with respect to x is _____

4. A linear system is _____ if it has no solution.

5. The rank of matrix A is the total number of non-zero_____ in reduced row echelon form.

6. A function $f(x) = \begin{cases} 3-x, & x < 2 \\ 2, & x = 2 \\ x/2, & x > 2 \end{cases}$ has _____ discontinuity at $x = 2$.

7. Equation of tangent to the curve $y = 4 - x^2$ at (-1, 3) is _____

8. The area bounded by x-axis and the graph of $f(x) = x^3, \ -1 \leq x \leq 1$ is _____.

9. The domain of a function $g(x) = 1 - \sqrt{x}$ is _____

10. The mean value theorem says that at some interior points of an interval, the instantaneous rate of change must equal _____

SECTION "B"

[10Q × 1=10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The area under the curve $y = \cos x$, x −axis from 0 to π equals _____
[0; 1/2; 1; 2]

12. $\int \tan u \ du =$ _____.
[$\ln|\cos u| + c$; $\ln|\sin u| + c$; $\sec^2 u + c$; $\ln|\sec u| + c$]

13. A function $y = f(x)$ is differentiable at _____.
[Corner; Cusp; Vertical tangent; Horizontal tangent]

14. If $s = -t^3 + 3t^2 - 3t$ represents the position of a body moving on a coordinate line then it's acceleration at $t = 0$ is _____.
[0; -3; 6; -6]

15. $\int \frac{ydy}{\sqrt{16-y^2}} =$_____.
[$\frac{1}{2\sqrt{16-y^2}} + C$; $\frac{1}{4}\sin^{-1}\left(\frac{y}{4}\right) + C$; $\sqrt{16-y^2} + C$; $-\sqrt{16-y^2} + C$]

16. The absolute maximum value of $f(x) = 4 - x^2, -3 \leq x \leq 1$ is _____
[0; 1; -3; 4]

17. If $f(x) = \lfloor x \rfloor$ represents integer floor function then $\lim_{x \to 3^-} \frac{\lfloor x \rfloor}{x} =$_____
[1; $\frac{2}{3}$; $\frac{3}{2}$; 3]

18. A linear system is called _____ if all of the constant terms are zero.
[consistent; inconsistent; homogeneous; non homogeneous]

19. If $\frac{dy}{dx}$ represents the derivative of y with respect to x, $\frac{d}{dx}(3^x) =$_____.
[3^{x-1}; $x3^{x-1}$; $3^x \ln 3$; 3^x]

20. Let $y = f(x)$ be given function and $c > 0$ be any constant. Then the function $y = f(cx)$ is obtained by _____.
[Stretching $y = f(x)$ horizontally by a factor c;
Stretching $y = f(x)$ vertically by a factor c;
Compressing $y = f(x)$ vertically by a factor c;
Compressing $y = f(x)$ horizontally by a factor c]

KATHMANDU UNIVERSITY
End Semester Examination [C]
November/December, 2023

30 NOV 2023

Level : B.Sc./B.Tech.
Year : I
Time : 2 hrs. 30 mins.

Course : MATH 111
Semester : I
F. M. : 55

SECTION "C"

[3 Q. × 7 = 21 marks]

1. Discuss the nature of the curve $y = -2x^3 + 6x^2 - 3$ on the basis of critical points, increasing/decreasing, points of inflection and concavity. Use this information to sketch the curve. [1+1.5+1+1.5+2]

OR

Discuss the curve $y = \frac{1}{x^2-1}$ with respect to asymptotes and concavity and sketch the curve. [3+2.5+1.5]

2. State first fundamental theorem of integral calculus. Find the area between the curves $x = y^2$ and $x = y^3$. Find average value of $f(x) = 4 - x^2$ on $[0, 3]$. [1+4+2]

3. Define the consistent and inconsistent of a system of linear equations. Test the consistency of the following system of linear equations and solve it: [2+5]

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8$$

SECTION "D"

[6 Q. × 4 = 24 marks]

4. Evaluate the following limits(*ANY TWO*):

a. $\lim_{y \to 0} \frac{5y^3+8y^2}{3y^4-16y^2}$ b. $\lim_{x \to 0} \frac{\sin x - x}{x^3}$ c. $\lim_{x \to 0^+} x^x$

5. Find $\frac{dy}{dx}$ (*ANY TWO*):

a. $y = \cos(x + \sqrt{x+1})$ b. $y = e^{(\cos x + \ln x)}$ c. $xe^y = x - y$

6. Evaluate the following integrals(*ANY TWO*):

a. $\int \frac{dx}{x^2+2x}$ b. $\int x^2 e^x \, dx$ c. $\int_0^2 \frac{2x}{x^2-5} dx$

7. Find all local extreme values of $f(x) = -x^3 + 2x^2$.

OR

Find absolute extreme values of $f(x) = xe^{-x^2/8}$ on $[-1, 4]$.

8. Solve the initial value problem: $\frac{dy}{dx} = \frac{x}{y^2}$, $y(2) = 3$.

9. Find the inverse of the matrix $\begin{bmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix}$, if it exists.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. For what value of a is $f(x) = \begin{cases} x^2 - 1 & x < 3 \\ 2ax & x \geq 3 \end{cases}$ continuous at every x.

11. Shift the graph $y^3 = x^2$, 2 units to the left, 2 units to the down. Find a single equation after shifting.

12. Find the linearization of $f(x) = \sqrt{1 + x}$ at $x = 3$.

13. Find the length of curve $y = x^2, -1 \leq x \leq 2$.

14. Find the row rank of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 7 & 8 \end{bmatrix}$.