KATHMANDU UNIVERSITY
End Semester Examination
July, 2024

Level : B.Arch.
Year : I
Time : 2 hrs. 30 mins.

18 JUL 2024

Course : MATH 105
Semester : I
F.M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define the derivative of a function at a point. How is it connected to the continuity of the function at the same point? Also, verify that the function $f(x) = x\,Sin(1/x)$ is continuous but not derivable at $x = 0$. [2+1+4]

2. Define a plane in 3-space and write its general equation. Also, find the equation of the plane through (3, 2, 1) and perpendicular to the line joining (-5, 3, 7) and (2, -4, 5). [2+1+4]

OR

What is meant by the co-planar lines? Prove that the lines $\frac{x+5}{3} = \frac{y+4}{-1} = \frac{z-7}{2}$ and $3x + 2y + z - 2 = 0 = x + 3y - 2z - 13$ are coplanar. Also, find the equation of the plane in which they lie. [1+3+3]

3. Write the necessary condition(s) for the function $f(x)$ to have local extreme value(s). Also, find all extreme values of the function $g(x) = -x^3 + 12x + 5$ defined on [-4, 4]. [2 + 5]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Find $\frac{dy}{dx}$ (*ANY TWO*):

(i) $y = \frac{x^2-2}{x^2+2}$ (ii) $x^3 - y^3 = 3xy$ (iii) $x = e^{\cos 2t}, y = e^{\sin 2t}$

5. Evaluate the following integrals (*ANY TWO*):

(i) $\int \frac{x+2}{\sqrt{x+3}}\,dx$ (ii) $\int_0^{\pi/2} x\,Cos\,x\,dx$ (iii) $\int (1+e^{\tan\theta})\sec^2\theta\,d\theta$

6. Find the asymptotes of the curve $y = \frac{x-1}{x+1}$ and sketch the curve.

7. Find the symmetrical form of the equation of straight line
$x + 2y - 3z = 4$; $2x + 3y - 4z = 2$.

8. Solve the differential equation: $2xy\,\frac{dy}{dx} = x^2 - 3y^2$ with $y(1) = 1$.

9. Sketch the graph and find the area bounded by the curves $y^2 = x$ and $y = x^2$.

OR

Evaluate the integral: $\int_0^{\infty} \frac{1}{4+x^2}\, dx$.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. For the function defined on $\Re^2$ by $f(x, y) = \ln(x^2 + y^2)$, find the first order partial derivatives.

11. Find the equation of the sphere with the line joining the points (2, 3, - 4) and (1, - 2, 3) as diameter.

12. Evaluate the limit: $\lim_{x \to 0} x^{1/x}$

13. Find the angle between the pair of planes: $x + 2y + z = 5$ and $2x + y - z = 2$.

14. Evaluate: $\Gamma\left(\frac{5}{2}\right) \Gamma\left(\frac{7}{2}\right)$.