KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

0 3 JUL 2023

Level : B. Arch.
Year : I
Time : 2 hrs. 30 mins.

Course : MATH 105
Semester: I
F.M. : 55

SECTION "C"
[3Q. × 7 = 21 marks]

1. Define the derivative of a function $y = f(x)$ at a point $x = a$. How is it related to the derivative of the function at that point? Also, show that the function $f(x) = x\,Sin\,(\frac{1}{x})$ is continuous but not derivable at $x = 0$. [2 +1+4]

2. Write the second derivative test for the extreme values of the function. Also, find the extreme values of the function $f(x) = -x^3 + 12\,x + 5$ defined on [-3, 3]. [2 +5]

OR

Define the first order partial derivatives of $f(x, y)$ relative to its variables. Also, use proper symbol to the find the mixed-order partial derivatives of the function $f(x,y) = tan^{-1}(xy)$. [2 + 5]

3. What do you mean by co-planar lines? Prove that the line $\boldsymbol{L}_1$: $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and line $\boldsymbol{L}_2$: $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Also, find the equation of the plane containing these lines. [1+3+3]

SECTION "D"
[6Q. × 4 = 24 marks]

4. Find $\frac{dy}{dx}$ (*ANY TWO*):

a. $x^3 + y^3 = 3xy$ b. $f(x) = \ln(\cos(e^{\sqrt{\sinh x}}))$ c. $x = e^t\,Sin\,t, y = e^t\,Cos\,t$

5. Evaluate the following integrals *ANY TWO*:

a. $\int e^{2x}\,Sin\,3x\,dx$ b. $\int_0^2 \frac{dx}{\sqrt[3]{(x-1)}}$ c. $\int_0^\infty \frac{1}{x^2+1}\,dx$

6. Define the gamma function and use it to evaluate: $\int_0^{\pi/2} Sin^4\theta\,Cos^6\theta\;d\theta$.

7. Find the equation of the plane passing through the points (4, 2, 3), (1, 2, 5) and (2, 6, 2).

OR

Find the equation of the line through (-1, 3, 2) that is perpendicular to the plane $x + 2y + 2z = 3$. Also, find the coordinate of its foot.

8. Sketch the graph and find the area bounded by the curves $y^2 = 4x$ and $4y = x^2$.

9. Solve the IVP: $x\,dy = (x^3 - y)\,dx \; with \; y(1) = 1$.

10. Find the equations of tangent and normal to the curve C: $x^2 + 3xy + y^2 = 5$ at (1, 1).

SECTION "E"

[5Q.× 2 = 10 marks]

10. Find the locus of points which are at a distance of 5 units from the point (1, - 2, 3).

11. Find the angle between the tangents to the curves $y^2 = x$ and $x^2 = y$ at their points of intersection other than origin.

12. Evaluate the limit: $\lim_{x \to 0} \left(\frac{1}{x}\right)^x$.

13. Find the direction cosines of the line joining points (1, 2, - 3) and (3, - 4, 5).

14. Find all asymptotes of the curve $f(x) = \frac{x^2 + 2}{x - 1}$.

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Marks Obtained:

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Course : MATH 105
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Exam Roll No.:

Time: 30 mins.

F.M. : 20

Registration No.:

Date 03 JUL 2023

SECTION "A"
[10Q. × 1 = 10 marks]

Fill in the blanks space(s) by most appropriate word(s) or symbol(s).

1. Every equation of first degree in x, y, z represents _____.

2. The range of the real function $f(x, y) =$ Sin (xy) is _____.

3. A line that meet a sphere in two coincident points is _____ to the sphere.

4. The derivative of $y =$ Cosh x is _____.

5. The graph of an even function $y = f(x)$ is symmetric about _____.

6. The circle with radius equal to radius of the sphere is called _____.

7. The necessary condition for the differential equation of the form $M(x, y)dx + N(x, y)dy = 0$ to be exact is _____.

8. The value of $\Gamma\left(\frac{7}{2}\right)$ is _____.

9. The first order differential equation of $x^2 + y^2 = 9$ is _____.

10. The direction cosines of a line parallel to y-axis is _____.

SECTION "B"
[10Q. × 1 = 10 marks]

Fill in the blank space (s). **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The z-intercept of the plane $2x - 3y + 4z = 12$ is _____.
[- 4; - 2 ; 3; 4]

12. $\lim_{x \to 2^-} \frac{|x-2|}{x-2} =$ _____
[-1; 0 ; 1; 2]

13. The average value of $g(x) = 4 + x^2$ on the closed interval [0, 3] is _____
[3 ; 5; 7; 9]

14. The maximum value of $h(x) = x^3 - 12x + 5$ is at _____.
[$x = -2$; $x = -1$; $x = 1$; $x = 2$]

15. The order and degree {order, degree} of the differential equation whose solution being $y = 2\cos x + 3\sin x$ is _____
[{1, 1}; {1, 2}; {2, 1}; {2, 2}]

16. The direction cosines to the normal to the plane $x - 2y + 2z = 3$ is _____
[$(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$; $(\frac{1}{3}, -\frac{2}{3}, \frac{2}{3})$; $(\frac{1}{3}, \frac{2}{3}, -\frac{2}{3})$; $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$]

17. The normal to the curve $x^2 + 3xy + y^2 = 5$ at (1, 1) is _____.
[-1; 0 ; 1; 2]

18. The value of $\frac{d^4}{dx^4}(Cos\ 2x)$ at $x = 0$ is _____.
[10; 12; 14 ; 16]

19. The value of $\int_0^3 (t^2 + 4)\, dt$ is .....
[18; 21; 24; 27]

20. The radius of the sphere $x^2 + y^2 + z^2 + 4x - 2y + 2z = 10$ is ..... units.
[2 ; 3; 4; 5]