KATHMANDU UNIVERSITY
End Semester Examination
February/March, 2018

Marks Obtained:

Level : B. Arch.
Year : I

Course : MATH 105
Semester: I

Exam Roll No.: Time: 30 mins. F.M. : 20

Registration No.: Date : MAR 0 8 2018

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blanks space(s) by most appropriate word(s) or symbol(s).

1. A function y = f(x) is even function if _____.

2. If $f: \Re \to \Re$. The domain of the function f(x) = $\sqrt{x+5}$ is _____.

3. The value of $\frac{d}{dx}\log(\tan x)$ = _____.

4. The slope of tangent to the curve $x^3 + y^3 = 9$ at (2, 1) is given by _____.

5. If $\lim_{x\to a^+} f(x) \neq \lim_{x\to a^-} f(x)$. Then function $y = f(x)$ has a _____ discontinuity at $x = a$.

6. If $F(x) = \int_0^x \sqrt[3]{t+5}\, dt$, the derivative of F(x) at x = 3 is _____.

7. $\lim_{x\to 3^-} \frac{\lceil x \rceil}{x}$ = _____.

8. The value of $\Gamma\left(\frac{5}{2}\right)$ = _____.

9. The first order differential equation of $x^2 + y^2 = 25$ is _____.

10. The perpendicular distance from origin to plane 2x – y + 2z = 5 is _____.

SECTION "B"
[10 Q. ×1 = 10 marks]

Fill in the blank space (s). DO NOT TICK, by selecting the most appropriate answers from among the given ones.

11 Two planes $a_1x + b_1y + c_1z = d_1$ and $a_2x + b_2y + c_2z = d_2$ will be parallel if _____.

[$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$; $a_1a_2 + b_1b_2 + c_1c_2 = 0$; $\frac{a_1}{a_2} = \frac{b_1}{b_2}$; $b_1b_2 + c_1c_2 = 0$]

12 A function f(x) will have a critical point at the point $x = a$ when _____.

[$f''(a) > 0$; $f''(a) = 0$; $f'(a) > 0$; $f'(a) = 0$]

13 The average value of f(x) = 4 – x^2 on the closed interval [0, 3] is _____.
[1/5; 2/5; 5; 1]

14. The value of f(x) = x^3 – 12x +5 is minimum when _____.
[$x = -2$; x = 2; $x = 1$; $x = -1$]

15. The value of $\Gamma(n + 1)$ is n! if n is _____.
[even number; real number; rational number; positive integer]

16. The graph of the function $x = y^2$ is symmetric about_____.
[y-axis; x-axis; line y = x; origin]

17. The vertical asymptote to the curve $y = \frac{x^2-3}{2x-6}$ is _____.
[$y = \frac{x}{2} + 1$; $y = \frac{x}{2} - 1$; $x = 3$; $y = 3$]

18. The value of $\frac{d^n}{dx^n}(e^{nx})$ is _____.
[$n^n e^{nx}$; $n!\, e^{nx}$; ne^{nx}; $n^n e^n$]

19. If $f'(x) = \frac{1}{1+x^2}$, $f(0) = 0$. Then f(x) = _____.
[$\sqrt{1-x^2}$; $\sqrt{1+x^2}$; $\tan^{-1}x$; $\sin^{-1}x$]

20. The value of $\int logx\, dx$ is _____.
[$\frac{1}{x} + c$; $xlogx - x + c$; $xlogx + x + c$; $\frac{1}{x}$]

KATHMANDU UNIVERSITY
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MAR 0 8 2018

Level : B. Arch.
Year : I
Time : 2 hrs. 30 mins.

Course : MATH 105
Semester: I
F.M. : 55

SECTION "C"
[3Q. × 7=21 marks]

1. Define the continuity of a function y = f(x) at a point x = a and write the types of discontinuity. Prove that differentiability of a function at a point implies continuity at that point but converse may not be true. [2 + 3 + 2]

2. Define critical point and point of inflection of a function. What are the critical points and point of inflection of the function $(x) = x^4 - 4x^3 + 10$? Also find local maxima and minima of this function. [2 + 2 + 3]

OR

State the Maximum and Minimum theorem of extreme value. What is right hand derivative and left hand derivative of a function at a point? For what value of a and b will f(x) = $\begin{cases} ax, & \text{if } x < 2 \\ ax^2 - bx + 3, & \text{if } x \geq 2 \end{cases}$ be differentiable for all values of x. Write down the necessary condition for local extreme value. [2 + 1 + 3 + 1]

3. Define the sphere and co-planar lines. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Find the equation of the plane in which they lie. [2+ 2+ 3]

SECTION "D"
[6Q. × 4 = 24 marks]

4. Find $\frac{dy}{dx}$ (*ANY TWO*):

(a) $y^x = x^y$ (b) $x^2 - 2xy + y^2 = 4$ (c) $x = e^{\log \cos 4\theta}$, $y = e^{\log \sin 4\theta}$

5. Evaluate the following integrals *(ANY TWO)*:

(i) $\int (e^x + x^3)\, dx$ (ii) $\int_0^2 \frac{dx}{\sqrt[3]{(x-1)}}$ (iii) $\int (1+e^{\tan\theta})\sec^2\theta\, d\theta$

6. Find the average value of f(x) = $x^2 + 2x + 2$ on [0, 3].

7. Find the area of the region between the curve y = $5x^2$ - 3x and the x- axis on the interval [0, 5] using limit of a sum.

OR

Find the length of the curve $y = \frac{3}{4}x^{\frac{4}{3}} - \frac{3}{8}x^{\frac{2}{3}} + 5$ from $x = 1$ to $x = 8$.

8. Define Beta and gamma function. Evaluate: $\Gamma\left(\frac{5}{2}\right)\Gamma\left(\frac{7}{2}\right)$.

9. Solve the differential equation $(2x - y)\, dx + (y - x)\, dy = 0$.

SECTION "E"

[5Q. × 2 = 10 marks]

10. Define $f: R^2 \to R^2$ defined by $f(x,y) = x^3 + x^2y^3 + 100$, find f_x and f_y where the symbols have their usual meanings.

11. Find the points on the curve $x^2 + y^2 = 9$ where the tangent is parallel x- axis.

12. Evaluate $\lim_{x \to 0} \frac{\cos x - e^x}{\sin x}$

13. Show that the plane $2x - y + 2z - 14 = 0$ touches the sphere $x^2 + y^2 + z^2 - 4x + 2y - 4 = 0$.

14. Find all asymptotes of the curve $f(x) = \frac{x^2 - x + 1}{x - 1}$.