

KATHMANDU UNIVERSITY
End Semester Examination
February/March, 2019

Marks Obtained:

Level : B.Arch.
Year : I

Course : MATH 105
Semester : I

Exam Roll No.:

Time: 30 mins.

F.M. : 20

Registration No.:

Date FEB 26 2019

SECTION "A"
[10 Q.×1=10 marks]

Fill in the blanks space(s) by most appropriate word(s) or symbol(s).

1. A function $y = f(x)$ is odd function if _____.
2. If $f : \mathcal{R} \rightarrow \mathcal{R}$. The domain of the function $f(x) = \frac{1}{x^2-9}$ is _____.
3. The value of $\frac{d}{dx} \log(\sin^2 x)$ _____.
4. The slope of tangent to the curve $2x^2 + 2xy = 4$ at $(1, 1)$ is given by _____.
5. If $\lim_{x \rightarrow 5^+} f(x) \neq \lim_{x \rightarrow 5^-} f(x)$, then function $y = f(x)$ has a _____ discontinuity at $x = 5$.
6. If $F(x) = \int_4^x \sqrt[3]{t+7} dt$, the derivative of $F(x)$ at $x = 20$ is _____.
7. $\lim_{x \rightarrow 5^+} \frac{[x]}{x} =$ _____.
8. The value of $\Gamma\left(\frac{1}{4}\right) \Gamma\left(\frac{3}{4}\right)$ _____.
9. The first order differential equation of $x^2 + y^2 = 25$ is _____.
10. The center of sphere $x^2 + y^2 + z^2 - 8x + 2y - 4 = 0$ is _____.

SECTION "B"
[10 Q. ×1 = 10 marks]

Fill in the blank space (s). DO NOT TICK, by selecting the most appropriate answers from among the given ones.

11. The graph of the function $y^2 = 4ax$ is symmetric about _____.
[y-axis; x-axis; line $y = x$; origin]

12. Two planes $a_1x + b_1y + c_1z = d_1$ and $a_2x + b_2y + c_2z = d_2$ will be perpendicular if

$$\left[\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}; \quad a_1a_2 + b_1b_2 + c_1c_2 = 0; \right.$$

$$\left. \frac{a_1}{a_2} = \frac{b_1}{b_2}; \quad b_1b_2 + c_1c_2 = 0 \right]$$

13. A function $f(x)$ will have a point of inflection at the point when _____
 $[f''(x) > 0; \quad f''(x) = 0; \quad f'(x) > 0; \quad f'(x) = 0]$

14. The average value of $\frac{1}{x}$ on the closed interval $[1, 3]$ is _____.
 $[1/2; \quad 2/3; \quad \ln 2/2; \quad \ln 3/2]$

15. The value of $f(x) = 4x^3 - 12x + 7$ is maximum when _____.
 $[x = -2; \quad x = 2; \quad x = 1; \quad x = -1]$

16. The value of $\Gamma(n)$ is $(n-1)!$ if n is _____.
 $[\text{even number}; \quad \text{real number}; \quad \text{rational number}; \quad \text{positive integer}]$

17. The oblique asymptote to the curve $y = \frac{x^2-3}{2x-4}$ is _____.
 $[y = \frac{x}{2} + 1; \quad y = \frac{x}{2} - 1; \quad y = -\frac{x}{2} + 1; \quad y = \frac{x}{2} + 2]$

18. The value of $\frac{d^n}{dx^n}(e^{nx})$ is _____.
 $[n^n e^{nx}; \quad n! e^{nx}; \quad ne^{nx}; \quad n^n e^n]$

19. If $f'(x) = \frac{1}{\sqrt{1-x^2}}$, $f(0) = 0$. Then $f(x) =$ _____.
 $[\sqrt{1-x^2}; \quad \sqrt{1+x^2}; \quad \tan^{-1}x; \quad \sin^{-1}x]$

20. If F and f are continuous function such that $F'(x) = f(x)$ for all x , then $\int_a^b f(x)dx$ is
 _____.
 $[F(b) - F(a); \quad F'(b) - F'(a); \quad f'(b) - f'(a); \quad f(b) - f(a)]$

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FEB 26 2019
Course : MATH 105
Semester: I
F.M. : 55

Level : B. Arch.
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Time : 2 hrs. 30 mins.

SECTION "C"

[3Q. × 7 = 21 marks]

1. Define the sphere and find the equation of sphere with center at origin and passing through (1, 2, 3). Write in a symmetrical form of the equation of a line of integration of the planes $x - 2y + 3z = 2$ and $3x - 4y + 5z = 8$. [1 + 2 + 4]
2. Define limit and continuity of a function $y=f(x)$ at a point $x=a$. Prove that differentiability of a function at a point implies continuity at that point but converse may not be true. [2 + 3 + 2]
3. Define the differential coefficient of a function $f(x)$ at a point $x = a$. What is right hand derivative and left hand derivative of a function at a point?
If $f(x) = \begin{cases} x, & \text{for } 0 < x < 1 \\ 2 - x, & \text{for } 1 \leq x \leq 2 \\ x - \frac{x^2}{2}, & \text{for } x > 2 \end{cases}$
Does $f'(x)$ exists at $x = 1$ and 2? [2 + 2 + 3]

OR

Define critical point and point of inflection of a function. Find local maxima and minima of function $f(x) = x^4 - 4x^3 + 10$. Also sketch the graph of function f . [2 + 2 + 3]

SECTION "D"

[6Q. × 4 = 24 marks]

4. Find $\frac{dy}{dx}$ (ANY TWO):
i) $y^{\sin y} = x^{\sin x}$ ii) $xy + y^2 = 1$ at (0, -1) iii) $x = e^{\cos 2t}, y = e^{\sin 2t}$
5. Evaluate the following integrals (ANY TWO):
i) $\int \operatorname{cosec}^3 x dx$ ii) $\int_0^{\pi/4} (1 + e^{\tan \theta}) \sec^2 \theta d\theta$ iii) $\int \frac{x^2 + 8}{x^2 - 5x + 6} dx$
6. Find the average value of $f(x) = \sin x$ on $[0, \frac{\pi}{2}]$ and verify the mean value theorem of average value.

7. Find the length of the curve $y = \frac{3}{4}x^{\frac{4}{3}} - \frac{3}{8}x^{\frac{2}{3}} + 5$ from $x = 1$ to $x = 8$

OR

Find the area of the region between the curve $y = x^4$ and $y = 8x$.

8. Define Beta and gamma function. Evaluate: $\Gamma\left(\frac{5}{2}\right) \Gamma\left(\frac{3}{2}\right)$.
9. Define exact differential equation and solve: $(x + y - 1)dx + (x - y - 2)dy = 0$.

SECTION "E"

[5Q. \times 2=10 marks]

10. Define $f: R^2 \rightarrow R$ defined by $f(x, y) = x^3 - 5x^2y + 2y^3 + 100$, find f_x and f_y , where the symbols have their usual meaning.
11. Find the points on the curve $x^2 + y^2 = 25$, where the tangent is parallel to y - axis.
12. Evaluate limit: $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$.
13. For what value of k will two planes $2x + 3y + 4z - 14 = 0$ and $2x - ky + 5z - 9 = 0$ be perpendicular.
14. Find all asymptotes of the curve $f(x) = \frac{2x}{x+1}$.