KATHMANDU UNIVERSITY
End Semester Examination
2024

Marks Scored:

Level : B.E./B.Sc.
Year : I

22 SEP 2024

Course : MATH 104
Semester : II

Exam Roll No.:

Time : 30 mins.

F.M. : 20

Registration No.:

Date : 22 SEP 2024

SECTION "A"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by most appropriate word(s) or symbol(s).

1. If the points $(r, \theta)$ and $(-r, \theta)$ both lie on the graph of a polar curve $r = f(\theta)$, then the curve in symmetric about_______________ .

2. If $\vec{u}$ is a differentiable vector function of $t$ of constant length, then the scalar product $\vec{u}.\frac{d\vec{u}}{dt} =$ ____________ .

3. If $w = f(x, y)$ is differentiable and $x, y$ are differentiable functions of $t$, then $w$ is a differentiable function of $t$ and by chain rule $\frac{dw}{dt} =$ ______________ .

4. A twice differentiable function $f(x, y)$ has a saddle point at a critical point $(a, b)$, then the value of discriminant $D = f_{xx}f_{yy} - f_{xy}^2$ at $(a, b)$ is _______________ .

5. The value of the double integral $\int_0^3 \int_0^2 (4 - y^2)\, dy\, dx =$ ______________ .

6. The Jacobian of the transformation $x = r\cos\theta, y = r\sin\theta$ is ______________ .

7. If $\vec{F}$ is a conservative field and $f$ is its potential function then the value of the integral $\int_A^B \vec{F}.d\vec{r} =$ _______________ .

8. The smallest period $T > 0$ of a periodic function is called ___________________ .

9. The curvature $\kappa$ of a straight line is ___________________ .

10. The equivalent Cartesian coordinates $(x, y, z)$ of the cylindrical coordinates $(r, \theta, z) = \left(\sqrt{2}, \frac{\pi}{4}, -1\right)$ is _______________ .

## SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), DO NOT TICK, by choosing the most appropriate answer from among the given ones.

11. The point $P\left(2, \frac{\pi}{6}\right)$ in polar coordinate system is equivalent to_______________ .

[$\left(2, -\frac{5\pi}{6}\right)$; $\left(-2, \frac{7\pi}{6}\right)$; $\left(-2, \frac{5\pi}{6}\right)$; $\left(2, -\frac{7\pi}{6}\right)$]

12. The polar equation $r = 6 \sin \theta$ represents a circle with center at__________ .

[$(3, 0)$; $(-3, 0)$; $(0, 3)$; $(0, -3)$]

13. The function $f(x, y) = \frac{1}{2}(x^2 + y^2)$ decreases most rapidly at the point $(1, 1)$ in the direction of ________________ .

[$\frac{1}{\sqrt{2}}(-\vec{\imath} - \vec{\jmath})$; $\frac{1}{\sqrt{2}}(\vec{\imath} + \vec{\jmath})$; $\frac{1}{\sqrt{2}}(-\vec{\imath} + \vec{\jmath})$; $\frac{1}{\sqrt{2}}(\vec{\imath} - \vec{\jmath})$]

14. The value of $\frac{\partial f}{\partial y}$ at $(4, -5)$ of a function $f(x, y) = x^2 + 3xy + y - 1$ is _______ .

[13; 7; -7; $-13$]

15. The unit tangent vector $\vec{T}$ and the principal normal vector $\vec{N}$ of a space curve $\vec{r}(t)$ are ____________ .

[parallel; orthogonal; dependent; non-coplanar]

16. The divergence of the vector field $\vec{F}(x, y) = (x^2 - y)\vec{\imath} + (xy - y^2)\,\vec{\jmath}$ is __________ .

[$y + 1$; $y - 1$; $3x - 2y$; $3x + 2y$]

17. $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$=__________, where the symbols have their usual meaning.

[$\pi$; $\sqrt{2}\,\pi$; $2\pi$ $\frac{2\pi}{\sqrt{3}}$] ]

18. The value of $B(5, 3)$=________, where $B(\cdot, \cdot)$ denotes the beta function.

[$\frac{1}{35}$; $\frac{1}{42}$; $\frac{1}{105}$; $\frac{1}{210}$]

19. The linearization of the function $f(x, y, z) = xy + yz + xz$ at the point (1,0,0) is $L(x, y, z)$=__________ .

[$x + y$; $y + z$; $x + z$; $x + y + z$]

20. For an odd function $f(x)$, the value of integral of $\int_{-\pi}^{\pi} f(x) \cos nx$ =__________ .

[$\pi$; $\pi/2$; $2\pi$; 0]

KATHMANDU UNIVERSITY
End Semester Examination
September 2024

Level : B.E./B.Sc.
Year : I
Time : 2 hrs. 30 mins.

22 SEP 2024

Course : MATH 104
Semester : II
F. M. : 55

SECTION "C"
[4Q. × 7 = 28 marks]

1. Identify the symmetries and sketch the polar curve $r = 1 + \cos\theta$. Find the area of the region bounded between the circle $r = 1$ and the cardioid $r = 1 + \cos\theta$. [1+3+3]

2. State second derivative test for local extreme values. Find all local maxima, local minima and saddle points of the function $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$. [2+ 5]

3. Define curvature and torsion of a smooth curve. Find $\vec{T}, \vec{N}, \vec{B}, \kappa$ and $\tau$ for the space curve $\vec{r}(t) = (3\sin t)\vec{i} + (3\cos t)\vec{j} + 4t\vec{k}$. [2+5]

4. State the tangential and the normal forms of Green's theorem and use them to find the circulation and flux of the vector field $\vec{F} = (y^2 - x^2)\vec{i} + (x^2 + y^2)\vec{j}$ on the triangle bounded by the lines $y = 0, x = 3, y = x$. [2+5]

**OR**

State divergence theorem and verify it for the vector field $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$. [2+5]

SECTION "D"
[9 Q. × 3 = 27 marks]

5. Write the equation of the cone $z = \sqrt{x^2 + y^2}$ into spherical coordinates system.

6. Find the directional derivative of a function $f(x, y, z) = x^2 + 2y^2 - 3z^2$ at the point $P_0(1, 1, 1)$ along the direction $\vec{u} = \vec{i} + \vec{j} + \vec{k}$.

**OR**

Find the maximum and minimum values of the function $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$ by using Lagrange multiplier method.

**P.T.O.**

7. Find the volume of the solid whose base is the region in the $xy$-plane that is bounded by the parabola $y = 4 - x^2$ and the line $y = 3x$, while the top of the solid is bounded by the plane $z = x + 4$.

8. By reversing the order of integration, evaluate the integral $\int_0^1 \int_2^{4-2x} dydx$.

9. Define Gamma function and prove that $\Gamma(n+1) = n\Gamma(n)$ for $n > 0$.

10. Write the acceleration $\vec{a}$ in the form $\vec{a} = a_T\vec{T} + a_N\vec{N}$ without finding the unit tangent vector $\vec{T}$ and principal normal vector $\vec{N}$ of the function $\vec{r}(t) = (t+1)\vec{i} + 2t\vec{j} + t^2\vec{k}$.

11. Find the work done by force $\vec{F} = 2y\vec{i} + 3x\vec{j} + (x+y)\vec{k}$ over the curve $\vec{r}(t) = (\cos t)\vec{i} + (\sin t)\vec{j} + (t/6)\vec{k}, \quad 0 \le t \le 2\pi$ in the direction of increasing $t$.

12. Find a potential function of the conservative vector field
$\vec{F} = (y+z)\vec{i} + (x+z)\vec{j} + (x+y)\vec{k}$.

13. Find the Fourier series expansion of the periodic function $f(x) = x$ on the interval $[-\pi, \pi]$