4KATHMANDU UNIVERSITY
End Semester Examination
September 2024

Marks Scored:

Level : B.E./B.Sc.
Year : I

Course : MATH 104
Semester : II

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : 05 SEP 2024

SECTION "A"
[10Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. In spherical coordinates, $x^2 + y^2 + (z-1)^2 = 1$ is written as _______________.

2. The polar equation of the line passing through the origin and making an angle θ_0 is ___________.

3. The derivative of $f(x,y) = x^2 sin2y$ at $\left(1, \frac{\pi}{2}\right)$ in the direction of $\vec{v} = 3\vec{\imath} - 4\vec{\jmath}$ is _______.

4. The critical point of $f(x,y) = x^2 + y^2$ is __________.

5. The equivalent polar form of the integral $\frac{1}{\pi a^2}\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\,dy\,dx$ is _____________.

6. Value of $B(2,3)$ is _________, where $B(\cdot,\ \cdot)$ denotes the beta function.

7. Velocity of a particle whose motion in space is given by the position vector $\vec{r}(t) = 2\ln(t+1)\vec{\imath} + \frac{t^2}{2}\vec{\jmath} + \frac{t^2}{2}\vec{k}$ at $t = 1$ is ___________.

8. The plane containing unit tangent vector, $\vec{T}$ and principal normal vector, $\vec{N}$ is called ______________ plane.

9. The curl of a vector field, $\vec{F} = (x^2 - y)\vec{\imath} + 4y\vec{\jmath}$ is _________________.

10. If $f(x)$ is an even function, then $\int_{-\pi}^{\pi} f(x)\sin x\ dx =$ _______________.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. Which polar coordinate labels the same as $\left(2, \frac{\pi}{6}\right)$? ______________.

[$\left(2, -\frac{5\pi}{6}\right)$, $\left(-2, \frac{\pi}{6}\right)$, $\left(2, \frac{7\pi}{6}\right)$, $\left(-2, \frac{-5\pi}{6}\right)$]

12. The equation of the circle passing through the origin, having a radius a, and centre on positive x-axis is ______________.

[$r = 2asin\theta$, $r = -2asin\theta$, $r = 2acos\theta$, $r = -2acos\theta$]

13. The equation for the tangent line to the given level curves $xy = -4$ at $(2, -2)$ is ____________.

[$y = x$, $y = -x$, $y = x + 4$, $y = x - 4$]

14. If a function $z = f(x, y)$ is differentiable function of x, y, and x, y are functions of t, then z is differentiable function of t and ____________________.

[$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$, $\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$,
$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$, $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$]

15. The triple integral value of $F(x, y, z) = xyz$ over the cube bounded by the coordinate planes $x = 1,\ y = 1,\ z = 1$ is____________.

[$\frac{1}{3}$, 1, 2, 3]

16. The value of $\frac{\Gamma\left(\frac{7}{2}\right)\Gamma\left(\frac{3}{2}\right)}{\Gamma(6)}$ is ______________, where $\Gamma(\cdot)$ denotes the gamma function.

[1, $\frac{\sqrt{\pi}}{6}$, $\frac{\pi}{6}$, $\frac{\pi}{128}$]

17. The value of f_{xy} at $(1, 1)$ of the function $f(x, y) = xy^2 + x^2y^3 + x^3y^4$ is ______.

[10, 12, 6, 20]

18. For any function $f(x, y, z)$ whose partial derivatives are continuous, then $\nabla \times \nabla f =$ ______________.

[-1, 0, ∇f, f]

19. Suppose the force field $\vec{F} = \nabla f$ is the gradient of the function $f(x, y, z) = \frac{1}{x^2+y^2+z^2}$. The work done by the force in moving an object along a smooth curve C joining $(1, 0, 0)$ to $(0, 0, 2)$ that does not pass through origin is

[$\frac{1}{4}$, $-\frac{1}{4}$ $-\frac{3}{4}$ $\frac{3}{4}$]

20. If $f(x)$ and $g(x)$ each have periods T, then the period of the function $h(x) = af(x) + bg(x)$ (a, b are constants) is __________________________.

[π, 2π, T, $2T$]