KATHMANDU UNIVERSITY
End Semester Examination [C]
January, 2019

Marks Scored:

Level : B.E./B.Sc./B. Tech.
Year : I

Course : MATH 104
Semester: II

Exam Roll No. :
Registration No.:

Time: 30 mins.

F. M. : 20
Date JAN 0 2 2019

SECTION "A"

[10 Q.× 1 = 10 marks]

Fill in the blank space (s) by most appropriate word (s) or symbol (s).

1. Cartesian form of the equation $r = 3\cos\theta$ is________________________.

2. Length of the polar curve $r = a$ is ________________.

3. The curvature of a straight line is ________________.

4. If $f(x, y) = x\cos y + ye^x$, then find $\frac{\partial^2 f}{\partial x \partial y} =$ ____________________.

5. The directional derivative of $f(x, y) = 2x + 3y$ at $(1, 1)$ in the direction of the vector $\vec{u} = \frac{1}{\sqrt{2}}\vec{\imath} + \frac{1}{\sqrt{2}}\vec{\jmath}$ is ____.

6. An equivalent integral of $\int_0^2 \int_{x^2}^{2x} dydx$ with the order reversed is __________________.

7. $\beta\left(\frac{1}{2}, \frac{1}{2}\right) =$ ________________, where $\beta(m, n)$ is the Beta function.

8. If $\vec{F}$ is conservative on D, then $\oint \vec{F}.d\vec{r} =$______________ around every simple closed loop in D.

9. ________________________ theorem generalizes the tangential form of Green's theorem from a flat surface in the plane to a surface in three dimensional space.

10. The primitive period of the function $f(x) = \tan(2x - 5)$ is______________________.

SECTION "B"

[10 Q.× 1 = 10 marks]

Fill in the blank space (s) by choosing the most appropriate answer from among the given ones. **Do not tick** the answers.

11. The polar equation $r^2 = 4\sin 2\theta$ represents ________________________________.
[Ellipse, Limacon, Lemniscate, Cardiod]

12. In Cylindrical Coordinate system (r, θ, z), the equation $z = 0$ represents________________.
[Pole , z-axis , Straight Line, Circle]

13. Level curve of the function $f(x, y) = 12 - x^2 - y^2$ that passes through the point(2, 2) is ______________________.

[$x^2 + y^2 = 4$, $x^2 + y^2 = 2$, $y^2 = x^2 - 8$, $x^2 = y^2 - 16$]

14. The expression $f_{xx}f_{yy} - f_{xy}^2$ is called ______________________ of f.

[Gaussian, Saddle, Hessian Eulerian]

15. $\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dydx =$ ______________________.

[0, a^2 πa^2, $2\pi a$]

16. For any function $f(x, y, z)$ whose second partial derivatives are continuous, curl grad $f =$ ______________________.

[-1, 0, 1, undefined]

17. Length of the curve $\vec{r}(t) = t\vec{i} + t\vec{j} + (1 + t)\vec{k}$ joining the two points $(0, 0, 1)$ and $(1, 1, 2)$ is ______________________.

[0, 1, $\sqrt{2}$, $\sqrt{3}$]

18. The divergence of vector field $\vec{F} = (x^2 - y)\vec{i} + (xy - y^2)\vec{j}$ is ______________________.

[$y + 1$, $x^2 + xy$, $3y - 2x$, $3x - 2y$]

19. The outward normal field of surface $x^2 + y^2 + z^2 = 4,\ z \geq 0$ is ______________________.

[$\vec{i} + \vec{j} + \vec{k}$, $\frac{x}{2}\vec{i} + \frac{y}{2}\vec{j} + \frac{z}{2}\vec{k}$, $x\vec{i} + y\vec{j} + z\vec{k}$, $2x\vec{i} + 2y\vec{j} + 2z\vec{k}$]

20. If $f(x)$ is an even function, then $\int_{-T/2}^{T/2} f(x)dx =$ ______________________.

[0, $\int_{-T}^{T} f(x)dx$, $2\int_{0}^{T/2} f(x)dx$, $4\int_{0}^{T/2} f(x)dx$]

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JAN 0 2 2019

Level : B.E./B.Sc./B. Tech.	Course : MATH 104
Year : I	Semester : II
Time : 2 hrs. 30 mins.	F. M. : 55

SECTION "C"

[4 Q.× 7 = 28 marks]

1. State Divergence theorem. Verify the theorem for the field $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$. Find the flux of $\vec{F} = xy\vec{i} + yz\vec{j} + zx\vec{k}$ outward through the surface of the cube cut from the first octant by the planes $x = 1, y = 1$ and $z = 1$. [1+4+2]

OR

Define conservative field? Write the component test criteria for a field to be conservative. Show that the field $\vec{F} = (y\sin z)\vec{i} + (x\sin z)\vec{j} + (xy\cos z)\vec{k}$ is conservative and find the potential function for the field. [1+1+5]

2. Describe the symmetry of polar curve about x –axis, y –axis and origin. Sketch the graph of the cardiod $r = 2 + 2\sin\theta$. Find the area of the surface generated by revolving the curve $r = \sqrt{\cos 2\theta}$ about y –axis. [2+3+2]

3. Derive tangential and normal components of acceleration of a particle moving along a curve in parametric form $\vec{r}(t)$. Without finding unit tangent vector $\vec{T}$ and unit normal vector $\vec{N}$, write the acceleration of the motion $\vec{r}(t) = (\cos t + t\sin t)\vec{i} + (\sin t - t\cos t)\,\vec{j}$ in the form $\vec{a} = a_T\vec{T} + a_N\vec{N}$, where a_T and a_N are tangential and normal components of acceleration. [4+3]

4. Define interior and boundary points of the region R. Find the absolute maximum and minimum values of the function $f(x,y) = 2 + 2x + 2y - x^2 - y^2$ on the triangular region in the first quadrant bounded by the lines $x = 0,\ y = 0,\ y = 9 - x$. [2+5]

SECTION "D"

[9 Q.× 3 = 27 marks]

5. Define Gamma function and evaluate $\int_0^a x^3(a^2 - x^2)^{5/2}dx$ using Beta and Gamma functions.

6. Determine Fourier coefficients a_0, a_n, b_n for the series

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx).$$

OR

Find the Fourier cosine series of $f(t) = t^2\ (0 < t < L)$.

7. Find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ if $z = 4e^x \ln y, x = \ln(r\cos\theta), y = r\sin\theta$ at $(r,\theta) = \left(2, \frac{\pi}{4}\right)$.

8. Estimate how much the value of $f(x,y,z) = y\sin x + 2yz$ will change if the point $P(x,y,z)$ moves 0.1 unit from $P_0(0,1,0)$ straight towards $P_1(2,2,-2)$.

OR

Determine the directions in which $f(x,y) = \frac{x^2}{2} + \frac{y^2}{2}$ increases and decreases most rapidly at $(1,1)$.

9. Integrate $\int_0^{2\pi}\int_0^{\pi}\int_0^{(1-\cos\phi)/2}\rho^2\sin\phi d\rho d\phi d\theta$.

OR

Convert the integral $\int_{-3}^{3}\int_0^{\sqrt{9-x^2}}\sin(x^2+y^2)dydx$ into the polar form and evaluate that integral.

10. Find the equations of tangent plane and normal line at the point $(1,-1,3)$ of the surface $f(x,y,z) = x^2+y^2+z^2+2xy-7=0$.

11. Find the area of the region shared by cardiod $r = 2(1-\cos\theta)$ and the circle $r = 2$.

12. Find κ and τ for the helix $\vec{r}(t) = (a\cos t)\vec{i} + (a\sin t)\vec{j} + (bt)\vec{k},\ a,b \geq 0,\ a^2+b^2 \neq 0$, where the symbols have their usual meanings.

13. Define circulation around a curve and find the circulation of the field $\vec{F} = (x-y)\,\vec{i} + x\,\vec{j}$ around the circle $x^2+y^2=1$.