KATHMANDU UNIVERSITY
End Semester Examination
May/June, 2022

Mark Scored:

Level : B.E./B.Sc./B.Tech.
Year : I

Course : MATH 104
Semester: II

Exam Roll No. :

Time: 30 mins.

F.M. : 20

Registration No.:

Date : May-30, 022

SECTION "A"
[10Q. × 1 = 10 marks]

Fill in the blank space (s) by most appropriate word (s) or symbol (s).

1. The center of the circle $r = 2\cos\theta$ is at ____________________.

2. The volume of the solid generated by revolving the curve $r = a$ about $x -$ axis is ____________.

3. The level curve of the function $f(x, y) = 16 - x^2 - y^2$ at $(2\sqrt{2},\ \sqrt{2})$ is ______________.

4. If the function $w = f(x, y)$ is differentiable and x, y are differentiable functions of t, then w is differentiable function of t and by chain rule, $\frac{dw}{dt} =$ ________.

5. $\int_1^2 \int_2^3 \int_3^4 dz\, dy\, dx =$ ____________________.

6. The value of $\Gamma\left(\frac{3}{2}\right) =$ ________________ where, the symbol $\Gamma(n)$ has its usual meaning.

7. The curvature of the straight line is, $\kappa =$ __________________.

8. The divergence of the vector field $\vec{F} = 2x\,\vec{i} + (x + 2y)\,\vec{j} + z\,\vec{k}$ is ________________.

9. The generalization of Green's theorem in three dimension in its normal form is known as $a(n)$ __________________ theorem.

10. For an odd function $f(x)$, $\int_{-T}^{T} f(x)dx =$ ________________.

SECTION "B"
[10Q. × 1 = 10 marks]

Fill in the blank space (s) by choosing the most appropriate answer from among the given ones.
DO NOT TICK the answers.

11. The eccentricity e of the conic section $r = \frac{2}{2 - 2\cos\theta}$ with one focus at origin is ______________.
[$e = 2$, $e = 1$, $e = \frac{1}{2}$, $e = \frac{1}{4}$]

12. The curve $r = 1 + \cos\theta$ is symmetric about ________________________.
[$x -$ axis, $y -$ axis, pole, pole and $x -$ axis]

13. In spherical coordinates $(\rho, \emptyset, \theta)$, the equation $\phi = \phi_o$ represents ______________________.
[Sphere centered at origin | Cone with vertex at origin
Cylinder about $z -$ axis | Straight line through origin]

14. If $w = x^2 + y - z + \sin t$ and $x + y = t$, find $\left(\frac{\partial w}{\partial x}\right)_{y,z}$ = ______________________.
[$2x$, 1, $2x + \cos(x + y)$, 0]

15. Beta function $B(m,n) =$ ______________________.
[$\int_0^1 x^{m-1}(1+x)^{n-1}dx$ | $2\int_0^{\pi/2} \cos^{2m-1} t \sin^{2n-1} t\, dt$
$2\int_0^{\pi/2} \cos^{2m-2} t \sin^{2n-2} t\, dt$, | $\int_0^{\pi/2} \cos^{2m-1} t \sin^{2n-1} t\, dt$]

16. The double integral ______________ gives the area enclosed by unit circle $x^2 + y^2 = 1$ in the first quadrant.

[$\frac{1}{2}\int_0^{\pi/2}\int_0^1 r^2 dr\, d\theta$, $\frac{1}{2}\int_0^{\pi/2}\int_0^1 r dr\, d\theta$, $\int_0^{\pi/2}\int_0^1 r dr\, d\theta$, $\int_0^{\pi/2}\int_0^1 dr\, d\theta$]

17. If $\vec{u}(t)$ is a differentiable function of t, of constant length, then ______________________.
[$\vec{u}$ is orthogonal to $\frac{d\vec{u}}{dt}$, | $\vec{u}$ is parallel to $\frac{d\vec{u}}{dt}$,
$\vec{u} \times \frac{d\vec{u}}{dt} = 0$, | $\vec{u} = \frac{d\vec{u}}{dt}$]

18. The relation ______________________ is not true.
[$\vec{B} = \vec{T} \times \vec{N}$, $\vec{N} = \frac{d\vec{T}/dt}{|d\vec{T}/dt|}$, $\kappa = \frac{1}{|\vec{v}|}\frac{d\vec{T}}{dt}$, $\rho\kappa = 1$]

19. If the field $\vec{F}$ is conservative on D, then ____________ for some scalar function f.
[$\vec{F} = \nabla f$, $\vec{F}.\nabla f$=0, $\vec{F} = \nabla \times \nabla f$ $\nabla \times \vec{F} = \nabla f$]

20. The primitive period of the function $f(x) = \sin mx$ is ______________________.
[2π, $2m\pi$, $\frac{2\pi}{m}$, $\frac{\pi}{m}$]

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Time : 2 hrs. 30 mins.	F.M. : 55

SECTION "C"
[4Q. × 7 = 28 marks]

1. Discuss the symmetry and sketch the graph of the polar curve $r = 3 + 2\cos\theta$. Find the area of the region inside the circle $r = 4\cos\theta$ and to the right of the vertical line $r = \sec\theta$. [4+3]

2. Define critical point and saddle point. State the second derivative test for evaluating the local maxima and local minima. Find local extreme values, if they exist, of the function $f(x,y) = x^3 - y^3 - 2xy + 6$. [2+1+4]

3. State Stronger form of the Fubini's Theorem. Sketch the region of integration of the integral $\int_0^1 \int_y^{\sqrt{y}} 12x\, dxdy$ and write an equivalent integral with the order of integration reversed. Evaluate the latter integral. [1+3+3]

4. Define vector field. State divergence theorem and verify the divergence theorem for the field $\vec{F} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$. [1+1+5]

OR

State the component test for the exactness of the differential $Mdx + Ndy + Pdz$. Show that the differential $2xy\,dx + (x^2 - z^2)\,dy - 2yz\,dz$ is exact and then Evaluate the integral $\int_{(0,0,0)}^{(1,2,3)} 2xy\,dx + (x^2 - z^2)\,dy + z\,dz$. [1+2+4]

SECTION "D"
[9Q. × 3 = 27 marks]

5. Find the length of the polar curve, $r = 1 - \cos\theta$.

6. Evaluate the limit (if exists) of the function $f(x,y) = \dfrac{x^4}{x^4+y^2}$ as (x,y) approaches $(0,0)$.

7. Find the directions in which the function $f(x,y) = x^2 + xy + y^2$ increases and decreases most rapidly at $(-1,1)$. Find the derivatives of the function in these directions.

OR

Use chain rule to evaluate $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ if $w = xy + yz + zx,\ x = u + v,\ y = u - v,\ z = uv$ at $(u,v) = (1,1)$.

8. Find the average value of the function $F(x,y,z) = x^2 + y^2 + z^2$ over the cube in the first octant bounded by the coordinate planes and the planes $x = 1, y = 1$ and $z = 1$.

OR

Evaluate the integral: $\int_0^{2\pi} \int_0^{\pi} \int_0^{\sqrt{(1-\cos\phi)/2}} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$.

9. Prove that $\frac{B(m,n+1)}{n} = \frac{B(m+1,n)}{m} = \frac{B(m,n)}{m+n}$, where $B(m,n)$ is a Beta function.

10. The vector $\vec{r}\,(t) = (t^2+1)\,\vec{i} + (2t-1)\,\vec{j}$ defines the position of a particle moving in the plane at time t. Find the particle's velocity, acceleration and direction of motion at the time $t = \frac{1}{2}$.

11. Find $\vec{T}$, $\vec{N}$, κ for the space curve $\vec{r}\,(t) = (\cos t + t\sin t)\,\vec{i} + (\sin\, t - t\cos t)\,\vec{j} + 3\,\vec{k}$ where the symbols have their usual meanings.

12. Find the circulation of the field $\vec{F} = (x-y)\,\vec{i} + x\,\vec{j}$ around the circle $x^2+y^2 = 9$.

13. Find the Fourier series expansion of the function $f(x) = x\ (-\pi < x < \pi)$.