SOE.

KATHMANDU UNIVERSITY
End Semester Examination
March, 2025

Marks Scored:

Level : B.E./BIT
Year : I

20 MAR 2025

Course : MATH 104
Semester : II

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date :

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The Cartesian form of the polar equation $r^2 \sin 2\theta = 2$ is _____.

2. The level curve of $f(x, y) = y - x^2$ at $(1, 1)$ is _____.

3. If $u(x, y) = e^x \sin y$, then $u_{xx} + u_{yy} =$ _____.

4. The area of a closed and bounded region R in polar coordinates plane is given by the double integral _____.

5. The value of $\Gamma\left(\frac{5}{2}\right) =$ _____, where the symbol has its usual meaning.

6. The radius of curvature of the curve, $\vec{r}(t) = (\cos t)\ \vec{i} + (\sin\ t)\ \vec{j}$ is _____.

7. The length of the indicated portion of the curve

   $\vec{r}\ (t) = (2\cos t)\ \vec{i} + (2\sin t)\ \vec{j} + \left(\sqrt{5}\ t\right)\vec{k}\ ,\ 0 \leq t \leq \pi$ is _____.

8. Stoke's theorem is a generalization of Green's theorem in _____ form to three dimensions.

9. The gradient field of the function $f(x, y, z) = xyz$ is _____.

10. A function $f(x)$ is said to be periodic if it is defined for all x, and there is some positive integer T such that _____ for all x.

SECTION "B"

[10 Q.× 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The curve $r^2 = 4\sin 2\theta$ is symmetrical about _____.

[$x-$ axis; $y-$ axis; origin; all]

12. The center of the circle $r = 2\sin\theta$ is at _____.

[$(0,0)$; $(1,\ 0)$; $\left(1,-\frac{\pi}{2}\right)$; $\left(1,\ \frac{\pi}{2}\right)$]

13. Tangent plane of the surface $f(x,y,z) = x^2 + y^2 + z$ at $(0,0,0)$ is _____.

[$x + y + z = 0$; $2x + 2y + 1 = 0$; $x + y = 0$; $z = 0$]

14. If $w = x^2 + 2y^3,\ x = \sin t,\ y = \cos t$, then $\frac{dw}{dt}$ at $t = \frac{\pi}{2}$ equals to _____.

[0; -6; 2; 6]

15. The Jacobian of the transformation $x = r\cos\theta,\ y = r\sin\theta$ is _____.

[r; r^2; $r\sin\theta\cos\theta$; $r^2\sin\theta\cos\theta$]

16. Let $\vec{B} = \vec{T} \times \vec{N}$. The torsion function of a smooth curve is $\tau =$ _____.

[$-\frac{d\vec{B}}{ds}.\vec{N}$; $-\frac{d\vec{B}}{ds}\times\vec{N}$; $-\frac{d\vec{N}}{ds}.\vec{B}$; $-\frac{d\vec{N}}{ds}\times\vec{B}$]

17. If the acceleration vector is written as $\vec{a} = a_T\,\vec{T} + a_N\,\vec{N}$, then $a_T =$ _____.

[$\frac{d^2s}{dt^2}$; $\kappa|\vec{v}|^2$; $\left(\frac{ds}{dt}\right)^2$; $\frac{d\vec{v}}{dt}$]

18. If a vector field, $\vec{F}$ is conservative then, $\vec{F}$ = _____, for some scalar function f.

[∇f; $\nabla.\nabla f$; $\nabla\times\nabla f$; f]

19. The formula for the flux of a vector field, $\vec{F}(x,y) = P(x,y)\,\vec{\imath} + Q(x,y)\,\vec{\jmath}$ across a simple closed curve C is _____.

[$\int_C Pdy - Qdx$; $\int_C Pdy + Qdx$; $\int_C Pdx - Qdy$; $\int_C Pdx + Qdy$]

20. The fundamental period of the function $y = \tan 2x$ is _____.

[$\frac{\pi}{4}$; $\frac{\pi}{2}$; π; 2π]

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Level : B.E./BIT
Year : I
Time : 2 hrs. 30 mins.

20 MAR 2025

Course : MATH 104
Semester : II
F. M. : 55

SECTION "C"
[4 Q. × 7 = 28 marks]

1. Illustrate the standard polar coordinate tests for symmetry. Check the symmetry and sketch the polar curve $r = 2 - 2\cos\theta$. Find the area of the region that lies inside the circle $r = 1$ and outside the cardiod $r = 1 - \cos\theta$. [2+3+2]

2. Explain directional derivative and discuss its properties. Find the derivative of the function $f(x, y) = x^3 - xy^2$ at $P_0(1, 1)$ in the direction of $\vec{v} = 3\vec{\imath} - 4\vec{\jmath}$. In what directions does this function f increase most rapidly and decrease most rapidly? [3+2+2]

3. Define unit tangent vector $(\vec{T})$, principal unit normal vector $(\vec{N})$ and curvature (κ). Find $\vec{T}$, $\vec{N}$, κ and ρ for the space curve $\vec{r}(t) = (3\sin t)\,\vec{\imath} + (3\cos t)\,\vec{\jmath} + 4t\,\vec{k}$, where the symbols have their usual meanings. [3+4]

4. Define conservative vector field and potential function. Verify whether the vector field $\vec{F} = (2xy)\,\vec{\imath} + (x^2 + z)\,\vec{\jmath} + (y)\,\vec{k}$ is conservative. If it is, find the potential function for this vector field. [2+2+3]

OR

Describe flux and divergence of the vector field $\vec{F}(x, y)$. State Green's theorem in tangential form. Verify this form for the field, $\vec{F} = -x^2 y\,\vec{\imath} + xy^2\vec{\jmath}$ and the region R bounded by the circle $\vec{r}(t) = (a\cos t)\,\vec{\imath} + (a\sin t)\,\vec{\jmath}$, $0 \le t \le 2\pi$. [2+1+4]

SECTION "D"
[9Q. × 3 = 27 marks]

5. Find the spherical and cylindrical coordinates of the point whose Cartesian coordinate is $(0, 1, 0)$.

6. Find the partial derivatives $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2$, $x = \frac{r}{s}$, $y = r^2 + \ln s$ and $z = 2r$.

OR

Find the limit of the function (if it exists) $f(x, y) = \frac{2x^2y}{x^4+y^2}$ as (x, y) approaches $(0, 0)$. Discuss the continuity of this function at the origin.

P.T.O.

7. Find the greatest and smallest values that the function $f(x, y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$.

8. State Gamma function and use it to evaluate $\int_0^{\pi/2} \cos^3\theta \sin^4\theta \, d\theta$.

9. Change the Cartesian integral $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) dx \, dy$ into an equivalent polar integral and then evaluate the polar integral.

OR

Evaluate the integral $\int_0^{2/3} \int_y^{2-2y} (x + 2y) e^{y-x} dx \, dy$ using the transformation

$u = x + 2y, v = x - y$.

10. Evaluate the triple integral, $\int_{-1}^1 \int_0^{2\pi} \int_0^{1+\cos\theta} 4r \, dr \, d\theta \, dz$.

11. Find the velocity, speed, and acceleration of a particle whose motion in space is given by the position vector $\vec{r}(t) = (6\cos t)\ \vec{i} + (6\sin t)\,\vec{j} + 8t\,\vec{k}$ at time t.

12. Evaluate $\int_C (x - 3y^2 + z)\, ds$ over the line segment C joining the origin to the point $(1, 1, 1)$.

13. Find the Fourier series of $f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$.