KATHMANDU UNIVERSITY
End Semester Examination
March/ April, 2017

MAR 2 6 2017

Level : B.Sc.
Year : I
Time : 2 hrs. 30 mins.

Course : MATH 104
Semester : II
F. M. : 55

SECTION "C"
[4 Q. × 7 = 28 marks]

1. Define Beta and Gamma functions. Prove the relation $B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, $m,n > 0$. [2 + 5]

OR

Prove the relation $\int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{\Gamma\left(\frac{p+q+2}{2}\right)}$. Use this relation to evaluate the integral $\int_0^{\pi/2} \sin^4\theta \cos^6\theta \, d\theta$. [3 + 4]

2. Discuss and sketch the cardioid $r = a(1+\cos\theta)$, $a > 0$, and find the area of the region inside this cardioid. [4 + 3]

3. Define directional derivative $\left(\frac{\partial f}{\partial h}\right)_{\vec{u}, P_0}$ of a function $f(x,y)$ at a point $P_0(x_0, y_0)$ in the direction of unit vector $\vec{u}$. Derive the relation $\left(\frac{\partial f}{\partial h}\right)_{\vec{u}, P_0} = \nabla f(P_0)\cdot\vec{u}$. Find the directional derivative of $f(x,y) = 2xy - 3y^2$, $P_0(5,5)$ and $\vec{u} = 4\vec{i} + 3\vec{j}$. [2 + 3 + 2]

4. Define curvature and radius of curvature of a curve. Find $\vec{T}$, $\vec{N}$, κ and ρ of the plane curve $\vec{r}(t) = t\,\vec{i} + (\log\cos t)\,\vec{j}$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$ where the symbols have their usual meanings. [2 + 5]

SECTION "D"
[9 Q. × 3 = 27 marks]

5. Evaluate $\int_0^{\pi/2}\int_0^{1}\int_{-2}^{3} \sin z \, dx\, dy\, dz$.

6. Find the Fourier series of $f(x) = x$, $-\pi \le x \le \pi$.

7. Use Beta and Gamma functions to evaluate the integral $\int_0^1 x^3(1-x)^2\, dx$.

8. Verify that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ for the function $f(x,y) = x\cos y + ye^x$.

9. Show that the vector field $\vec{F}(x,y) = \frac{2x}{y}\vec{i} + \left(\frac{1-x^2}{y^2}\right)\vec{j}$ is conservative.

OR

Find the work done by $\vec{F}(x,y,z) = 6z\vec{i} + y\vec{j} + 12x\vec{k}$ over the curve $\vec{r}(t) = (\sin t)\vec{i} + (\cos t)\vec{j} + \left(\frac{t}{6}\right)\vec{k}, 0 \le t \le 2\pi$.

10. Find the center and radius of the circle $r = 2a\cos\theta$, where a is a constant.

11. If $\vec{u}$ and $\vec{v}$ be differentiable vector functions of t, then show that $\frac{d}{dt}(\vec{u}\cdot\vec{v}) = \frac{d\vec{u}}{dt}\cdot\vec{v} + \vec{u}\cdot\frac{d\vec{v}}{dt}$.

12. Change the Cartesian integral $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy\,dx$ into polar integral, and then evaluate the integral.

13. Show that the function $f(x,y) = \begin{cases} \frac{x^2 y}{x^2+y^2}, & (x,y) \ne (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ is continuous at origin.

Marks Scored:

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SECTION "A"

[10Q × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The Gamma function $\Gamma(n)=$ ______________________________ if n is zero or a negative integer.

2. The series $f(x)=\sum_{n=1}^{\infty} b_n \sin nx$ is a Fourier series of an ____________________ function $f(x)$.

3. The field $\vec{F}(x,y)=x\vec{i}+y\vec{j}$ is conservative in a domain $D\subset\mathbb{R}^2$, and consider a simple closed curve C within D. Then $\oint_C \vec{F}\cdot d\vec{r} =$ ____________________, where $\vec{r}(t)=x(t)\vec{i}+y(t)\vec{j}$ is the parametric representation of the curve C.

4. The unit tangent vector of the curve $\vec{r}(t)=(\cos t)\vec{i}+(\sin t)\vec{j}$ is ____________________.

5. The polar equation for the circle $x^2+(y-3)^2=9$ is ____________________.

6. $\lim_{(x,y)\to(0,0)} \dfrac{x^2-xy}{\sqrt{x}-\sqrt{y}}=$ ____________________.

7. $\int_0^1\int_0^{\pi} dy\,dx=$ ____________________.

8. The arc length of a curve $r=\cos\theta,\ 0\le\theta\le\dfrac{\pi}{2}$ is ____________________.

9. If $f(x,y)=\tan^{-1}\left(\dfrac{2xy}{x^2-y^2}\right)$, then $f_x=$ ____________________.

10. The spherical equation of the cylindrical equation $r^2+z^2=4$ is ____________________.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), DO NOT TICK, by selecting the most appropriate answer from among the given ones.

11. The polar equation $r=\dfrac{ke}{1-e\cos\theta}$ of the conic section represents a parabola if __________.

[$e=1$; $k=1$; $e>1$; $k<1$]

12. The mass of a thin plate of constant density $\rho(x,y)=3$ bounded by a rectangle $0 \le x \le 1,\ 0 \le y \le 1$ is ________________.

[1; e; 3; π]

13. $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)=$ ________________, where the symbols have their usual meanings.

[$\sqrt{2}$; $\sqrt{\pi}$; $\sqrt{2\pi}$; $\sqrt{2}\,\pi$]

14. The line integral $\int_C ds =$ ________________ where $C: x^2+y^2=1$.

[0; π; 2π; $\pi-1$]

15. $\int_0^1\int_0^1 x\,dx\,dy =$ ________________

[0; $\frac{1}{2}$; 1; $\frac{3}{2}$]

16. If a differentiable function $f(x,y)$ has a constant value along a smooth curve $C: \vec{r}(t)=x(t)\vec{i}+y(t)\vec{j}$, then $\nabla f \cdot \frac{d\vec{r}}{dt} =$ ________________.

[0; $\frac{\pi}{4}$; $\frac{\pi}{2}$; π]

17. The Cartesian coordinates (x,y,z) of the cylindrical coordinates $(r,\theta,z)=(1,0,0)$ is ________________, where $*$ is any arbitrary value.

[$(1,*,0)$; $(1,0,0)$; $(1,0,*)$; $(0,1,0)$]

18. The inequality $1-\frac{x^2y^2}{3} < \frac{\tan^{-1} xy}{xy} < 1$ tells us $\lim_{(x,y)\to(0,0)} \frac{\tan^{-1} xy}{xy} =$ ________________.

[0; $-\frac{1}{3}$; $\frac{1}{3}$; 1]

19. The function $f(t)=\begin{cases} -1, & -1<t<0 \\ 1, & 0<t<1 \end{cases}$ is ________________.

[even; odd; neither; both]

20. The area of the region between the origin and the curve $r=f(\theta), \alpha \le \theta \le \beta$ is given by ________________.

[$\int_\alpha^\beta r\,d\theta$; $\int_\alpha^\beta r^2\,d\theta$; $\frac{1}{2}\int_\alpha^\beta r\,d\theta$; $\frac{1}{2}\int_\alpha^\beta r^2\,d\theta$]