KATHMANDU UNIVERSITY
End Semester Examination [C]
June/July 2024

Level : B.E./B.Sc./B.Tech.
Year : I
Time : 2 hrs. 30 mins.

02 JUL 2024

Course : MATH 104
Semester : II
F. M. : 55

SECTION "C"
[4 Q. × 7 = 28 marks]

1. Discuss the symmetry of a polar curve $r = f(\theta)$ about x-axis, y-axis, and origin. Sketch the graph of the polar curve $r = 1 - \cos\theta$. Find the area shared by the circles $r = 2\cos\theta$ and $r = 2\sin\theta$. [2+3+2=7]

2. State the Second Derivative Test for evaluating local maxima, local minima, critical points, and saddle points for a function $f(x, y)$. Use this test to find the local maximum and minimum values of $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$. [3+4=7]

3. Define the exact differential form. State the component test to show the exactness of the differential form $Mdx + Ndy + Pdz$. Show that $(y + z)\,dx + (z + x)\,dy + (x + y)\,dz$ is exact. Evaluate the integral $\int_0^1\int_0^1\int_0^1 (y + z)\,dx + (z + x)\,dy + (x + y)\,dz$. [1+1+2+3=7]

OR

Define flux, divergence, and orientable surface. State the divergence theorem and verify the theorem for the expanding vector field $\vec{F} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$ over the sphere $x^2 + y^2 + z^2 = 9$. [3+1+3=7]

4. Find the average value of the function $f(x, y) = \sin(x + y)$ over the rectangle $0 \le x \le \pi$, $0 \le y \le \frac{\pi}{2}$. Sketch the region of integration and write an equivalent integral of $\int_0^1\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y\,dx\,dy$ with the order of integration reversed and then evaluate the integral. [2+2+3=7]

SECTION "D"
[9 Q. × 3 = 27 marks]

5. Use two path tests to check the continuity of the function

$$f(x, y) = \begin{cases} \frac{x^2 - y^2}{x^2 + y^2}, & (x, y) \neq (0,0) \\ 0, & (x, y) = (0,0) \end{cases}$$ at the origin.

6. Find the Cartesian and cylindrical coordinate equations for the equation $\phi = 5\pi/6$, $0 \le \rho \le 2$ with proper ranges for z in Cartesian and r in cylindrical coordinates.

P.T.O.

7. Find the derivative of $f(x,y) = x^2 + xy$ at $P_0(1,2)$ in the direction of $\vec{u} = \vec{i} + \vec{j}$.

OR

Use chain rule to evaluate $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ at $(r,s) = (1,1)$ if $w = x + 2y + z^2,\ x = r + s,$ $y = r^2 + \ln s, z = 2r$.

8. Evaluate the integral $\int_0^{2\pi}\int_0^3\int_0^{z/3} r^3 dr\, dz\, d\theta$.

OR

Evaluate $\int_0^4\int_{x=y/2}^{x=y/2+1} \frac{2x-y}{2}\, dx\, dy$ by applying the transformation $u = \frac{2x-y}{2}, v = \frac{y}{2}$ and integrating over an appropriate region in uv-plane.

9. Show that $\beta(m,n) = \frac{n-1}{m+n-1}\beta(m,n-1) = \frac{m-1}{m+n-1}\beta(m-1,n)$, where $\beta(m,n)$ represents a beta function.

10. Find $\vec{T}, \vec{N}$, and κ for the space curve $\vec{r}(t) = (3\sin t)\vec{i} + (3\cos\ t)\,\vec{j} + 4t\,\vec{k}$, where the symbols have their usual meanings.

11. Find the velocity, speed, and acceleration of the particle at the time $t = 1$ if the position of the particle at time t is given by $\vec{r}(t) = (t^2+1)\ \vec{i} + (2t-1)\,\vec{j} + 3\,\vec{k}$.

12. Find the flux of the vector field $\vec{F} = 2x\,\vec{i} - 3y\,\vec{j}$ across the circle $x^2 + y^2 = 1$.

13. Find the Fourier series expansion of $f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$.

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Registration No.: Date : 0 2 JUL 2024

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The equation $r = 2 + 3\cos\theta$ represents a polar curve known as______________.

2. The Cartesian form of the equation $r = \frac{4}{2\cos\theta - \sin\theta}$ is ___________.

3. The Hessian of the function $f(x, y) = x^2 + y^2$ at $(1, 2)$ is _______________.

4. If $f(x, y, z) = x + 2y - 3z$, then $\nabla f =$ _______________.

5. For the transformation $x = u\cos v,\ \ y = u\sin v$, Jacobian $J(u, v) =$ __________.

6. Let $f(x, y)$ be continuous on a region R. If R is defined by $a \le x \le b,\ g_1(x) \le y \le g_2(x)$, with g_1 and g_2 continuous on $[a, b]$, then $\iint_R f(x, y)dA =$ ________________.

7. The value of $\Gamma(5) =$ ___________________, where the symbol has its usual meaning.

8. If $\vec{r}(t)$ is a differentiable vector function of t of constant length, then $\vec{r}.\frac{d\vec{r}}{dt} =$ ________.

9. _______________ theorem generalizes Green's theorem in tangential form to three dimensions.

10. The divergence of a vector field, $\vec{F} = 2z^2\,\vec{i} - 3y^2\vec{j} + 4x^2\,\vec{k}$ is __________________.

SECTION "B"
[10 Q.× 1 = 10 marks]

Fill in the blank space (s) by choosing the most appropriate answer from among the given ones. **DO NOT TICK** the answers.

11. The polar coordinate of the rectangular coordinate point $(0, 1)$ is _______________.

    [$(1,\ \pi)$, $\left(1, \frac{\pi}{2}\right)$, $\left(1,\ \frac{3\pi}{2}\right)$, $\left(0,\ \frac{\pi}{3}\right)$]

12. The equation $r = 2\cos\theta$ represents a circle whose center is located at______________.

[$x -$ axis, $y -$ axis, origin, $(2,2)$]

13. $\lim_{(x,y)\to(2,2)} \frac{x+y-4}{\sqrt{x+y}-2} =$________.

[0, 4, 6, not defined]

14. The derivative of a function $z = f(x,y)$ in the direction where it rises most rapidly is ______.

[∇f, $-\nabla f$, $|\nabla f|$, $-|\nabla f|$]

15. If a function $z = f(x,y)$ is a differentiable function of x, y, and x, y are functions of t, then z is a differentiable function of t and _____________.

[$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$, $\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$,

$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$, $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$]

16. The double integral _____________ computes the area of a closed and bounded region R in the plane, which is enclosed by the curves $y = x$ and $y = x^2$.

[$\int_0^1 \int_{x^2}^x dx\, dy$, $\int_0^1 \int_{x^2}^x dy\, dx$, $\frac{1}{2}\int_0^1 \int_{x^2}^x dy\, dx$, $\int_0^x \int_x^{x^2} dx\, dy$]

17. The relationship between Beta and Gamma functions is expressed as $\beta(x,y) =$_________.

[$\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$, $\frac{\Gamma(x)\,\Gamma(y)}{\Gamma(x)+\Gamma(y)}$, $\frac{\Gamma(x+y)}{\Gamma(x)\Gamma(y)}$, $\frac{1}{2}\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$]

18. The plane bounded by unit tangent vector and principal unit normal vector is called ____________.

[normal plane, rectifying plane, osculating plane, principal plane]

19. For any function $f(x,y,z)$ whose partial derivatives are continuous, $\nabla \times \nabla f =$ _______.

[-1, 0, ∇f, f]

20. If the function $f(x)$ is ___________, then $\int_{-\pi}^{\pi} f(x)dx = 0$.

[even, odd, increasing, decreasing]