KATHMANDU UNIVERSITY
End Semester Examination [C]
January 2025

Marks Scored:

Level : B.E./B.Sc./B.Tech.
Year : I

Course : MATH 104
Semester : II

Exam Roll No.:
Registration No.:

Time: 30 minutes

F.M.: 20
Date: 10 JAN 2025

SECTION "A"
[10 Q.× 1 = 10 marks]

Fill in the blank space (s) by the most appropriate word (s) or symbol (s).

1. The polar equation $r = a + b\cos\theta$ represents the curve named as_______________ if $|a| = |b|$.

2. The value of the $\lim\limits_{(x,y)\to(1,1)} \frac{x^2-xy}{x-y} =$_____________.

3. If $f(x,y) = (x-1)(y+1)$, then the partial derivative $\frac{\partial f}{\partial x} =$ ___________.

4. A twice differential function $f(x,y)$ has a local maximum at (a,b) if $f_{xx} < 0$ and _____________ at (a,b).

5. According to the Fubini's theorem, if a function $f(x,y)$ is continuous on a region R defined by $a \le x \le b$, $h_1(x) \le y \le h_2(x)$ with h_1, h_2 continuous on $[a,b]$, then $\iint_R f(x,y)dA =$_____________.

6. Let $\vec{B} = \vec{T} \times \vec{N}$. The torsion of a smooth curve $\vec{r}(t)$ is given by the formula $\tau =$ _____________, where the symbols have their usual meanings.

7. The work done by a conservative force $\vec{F} = yz\,\vec{i} + xz\,\vec{j} + xy\,\vec{k} = \nabla f$, where $f(x,y,z) = xyz$ along a smooth curve joining the points $(-1,3,9)$ to $(1,6,-4)$ is _________________.

8. The divergence of the vector field $\vec{F} = y\,\vec{i} + (3x+y)\vec{j} - z\,\vec{k}$ is _________________.

9. Stoke's Theorem is the generalization of Green's theorem in _______________form in three dimensions space.

10. The fundamental period of the function $f(x) = \cos\frac{x}{2}$ is_________________.

SECTION "B"

[10 Q.× 1 = 10 marks]

Fill in the blank space (s) by choosing the most appropriate answer from among the given ones. **Do not tick** the answers.

11. The radius of the circle $r = 2\sin\,\theta$ is ______________.

    [1/2, 1, 2, 4]

12. The graph of Limacon $r = 2 + 3\sin\theta$ is symmetrical about______________.

    [$x -$ axis, $y -$ axis, pole, the line $y = x$]

13. In the spherical coordinates system, the equation $\phi = \frac{\pi}{3}$ represents the ______________________.

    [sphere centered at origin, cone with vertex at the origin, cylinder about $z -$ axis, straight line through origin]

14. If $w = x - y$ and $x = 2t,\ y = 2t^2$, then $\frac{dw}{dt} =$ ______________________at $t = 1$.

    [0, 1, 2, −2]

15. The value of $\Gamma\left(\frac{3}{2}\right) =$ ______________________, where $\Gamma(\cdot)$ denotes the gamma function.

    [$\frac{\sqrt{\pi}}{2}$, $\frac{\pi}{2}$, $\sqrt{\pi}$, $\sqrt{\frac{\pi}{2}}$]

16. The area of the region enclosed between the curves $y = x$ and $y = x^2$ is given by the double integral______________.

    [$\int_{-1}^{1}\int_{x^2}^{x} dydx$, $\int_{-1}^{1}\int_{x^2}^{x} dxdy$, $\int_{0}^{1}\int_{x^2}^{x} dydx$, $\int_{0}^{1}\int_{x^2}^{x} dxdy$]

17. The Jacobian $J(r,\theta)$ of the transformation $x = r\cos\theta,\ y = r\sin\theta$ is ______________.

    [$\sin\theta$, $\cos\theta$, r^2, r]

18. The length of the curve $\vec{r}\,(t) = 2\cos t\ \vec{\imath} + 2\sin\ t\ \vec{\jmath},\ 0 \le t \le 2\pi$ is______________.

    [π, 2π, $2\sqrt{2}\pi$, 4π]

19. If the acceleration vector is written as $\vec{a} = a_T\,\vec{T} + a_N\,\vec{N}$, then $a_T =$ ________________ where the symbols have their usual meanings.

    [$\frac{d\vec{v}}{dt}$, $\frac{d|\vec{v}|}{dt}$, $\frac{1}{|t|}\frac{d|\vec{v}|}{dt}$, $\frac{1}{|\vec{v}|}\frac{d|\vec{v}|}{dt}$]

20. If the vector field $\vec{F} = x\,\vec{\imath} - z\,\vec{\jmath} - y\,\vec{k}$ is conservative with its potential function $f(x,y,z)$, then $\nabla f =$ ________________.

    [$x\,\vec{\imath} - z\,\vec{\jmath} - y\,\vec{k}$, $\vec{\imath}$, $\vec{\imath} - \vec{\jmath} - \vec{k}$, $\vec{0}$]

KATHMANDU UNIVERSITY
End Semester Examination [C]
January 2025.

Marks Scored:

Level : B.E./B.Sc./B.Tech.
Year : I
Time : 2 hrs. 30 mins.

10 JAN 2025

Course : MATH 104
Semester : II
F.M. : 55

SECTION "C"

[4 Q.× 7 = 28 marks]

1. Discuss the symmetry test for the polar graphs. Sketch the graph of the curve $r = 1 - \cos\theta$. Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos\theta$. [2+3+2]

2. Define the directional derivative of a function $f(x, y)$ at point a (x_0, y_0) and explain how gradient vector of f relates with directional derivative. Find the directions in which the function $f(x,y) = \left(\frac{x^2}{2}\right) + \left(\frac{y^2}{2}\right)$ increases most rapidly, decreases most rapidly, and the direction of zero change of f at $(1, 1)$. Find the derivatives in these directions. [3+3+1]

3. Change the Cartesian integral $\int_0^1 \int_0^{\sqrt{1-y^2}} \sqrt{x^2 + y^2}\, dx\, dy$ into an equivalent polar form and then evaluate the polar integral. Find the average value of $f(x, y, z) = xyz$ on the cubical region bounded by the coordinate planes $x = 2, y = 2, z = 2$ in the first octant. [2+3+2]

4. Define circulation and flux on a simple closed curve. State the tangential form of the Green's Theorem. Verify this theorem for the vector field $\vec{F} = (x - y)\,\vec{i} + x\,\vec{j}$ in the region R bounded by the unit circle $C: \vec{r}(t) = \cos\, t\ \vec{i} + \sin\, t\,\vec{j},\ 0 \le t \le 2\pi$. [2+1+4]

OR

Define a potential function. Write the component test for a vector field $\vec{F} = M(x,y,z)\vec{i} + N(x,y,z)\,\vec{j} + P(x,y,z)\,\vec{k}$ to be conservative. Show that the vector field $\vec{F} = (y + z)\,\vec{i} + (x + z)\,\vec{j} + (x + y)\,\vec{k}$ is conservative and find a potential function for it. [1+2+4]

SECTION "D"

[9 Q.× 3 = 27 marks]

5. Express the equation of the sphere $x^2 + y^2 + (z - 1)^2 = 1$ into cylindrical and spherical coordinates systems.

6. Show that the function $f(x,y) = \frac{2x^2y}{x^4+y^2}$ has no limit as (x, y) approaches $(0, 0)$.

P.T.O.

7. Find the smallest and greatest values the function $f(x, y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$ using the method of Lagrange multiplier.

OR

Use chain rule to evaluate $\frac{\partial F}{\partial u}$ and $\frac{\partial F}{\partial v}$ at $(u, v) = (1, 1)$ if $F(x, y, z) = xy + yz + zx$ where $x = u + v,\ y = u^2 + 2v,\ z = 2u - v$.

8. Evaluate the integral $\int_0^{2/3} \int_y^{2-2y} (x + 2y)e^{y-x} dx\, dy$ by using the transformation $u = x + 2y,\ v = x - y$ writing it as the integral over the region in uv plane.

OR

Evaluate the triple integral $\int_0^{2\pi} \int_0^{\pi} \int_0^{\sqrt{(1-\cos\phi)/2}} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$.

9. Find the value of the integral $\int_0^1 x^{3/2}(1 - x)^{5/2}\, dx$ by using the Beta and Gamma functions.

10. If $\vec{r}\,(t) = (2 - t)\,\vec{\imath} + (3t^2 + 1)\,\vec{\jmath} + 4\,t^3\,\vec{k}$ defines the position of a moving particle at time t. Find the particle's velocity, acceleration, and direction of the motion at the time $t = 1$.

11. Find the unit tangent vector, principal unit normal vector, and the curvature of the space curve $\vec{r}\,(t) = (2\cos\, t)\,\vec{\imath} + (3\cos t)\,\vec{\jmath} + 4t\,\vec{k}$.

12. Evaluate the line integral $\int_C \vec{F}.\vec{T}\, ds$ of the vector field $\vec{F} = x^2\,\vec{\imath} - y\,\vec{\jmath}$ along the curve C: $y = x^2$ from $(0, 0)$ to $(2, 4)$.

13. Write the Fourier series expansion of the periodic function $f(x) = \begin{cases} 0, & -\pi < x < -\frac{\pi}{2} \\ 2, & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < x < \pi \end{cases}$.