KATHMANDU UNIVERSITY
End Semester Examination
August, 2019

Mark Scored:

Level : B.E./B.Sc./B.Tech.
Year : I
Exam Roll No. :
Registration No.:

Course : MATH 104
Semester: II
Time: 30 mins.
F.M. : 20
Date AUG 2 0 2019

SECTION "A"

[10Q × 1 = 10 marks]

Fill in the blank space (s) by most appropriate word (s) or symbol (s).

1. The eccentricity of the ellipse $r = \frac{6}{3+2\sin\theta}$ is $e =$__________.

2. Spherical coordinate equation of the cone $z = \sqrt{x^2 + y^2}$ is ________________.

3. At the point where $\kappa \neq 0$, the principal unit normal vector for a smooth curve in the plane is defined as $\vec{N} =$ ________________.

4. $\lim\limits_{(x,y)\to(0,0)} \frac{x^2-xy}{\sqrt{x}-\sqrt{y}} =$ __________.

5. The function $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ increases most rapidly at $(1, 1)$ in the direction $\vec{u} =$________.

6. The equivalent polar integral of the integral $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} dydx$ is ________________.

7. The value of $B(1, 2) =$ ________________ where, the symbol $B(m, n)$ has its usual meaning.

8. The speed of the particle at the given value of t, whose motion in space is given by $\vec{r}(t) = (2\cos t)\vec{i} + (3\sin t)\vec{j} + 4t\vec{k}$ at $t = \frac{\pi}{2}$ is ____________.

9. ____________ theorem is the generalization of Green's theorem in tangential form to three dimensions.

10. The Fourier coefficient a_n of a function $f(x)$ of period 2π is defined by the formula $a_n =$ ________________.

SECTION "B"

[10Q × 1 = 10 marks]

Fill in the blank space (s) by choosing the most appropriate answer from among the given ones. **DO NOT TICK** the answers.

11. The point $(-2, \frac{\pi}{6})$ labels the same as the point ____________________.

[$(-2, -\frac{\pi}{6})$, $(2, -\frac{\pi}{6})$, $(-2, \frac{7\pi}{6})$, $(2, \frac{7\pi}{6})$]

12. $r = 6\cos\theta$ represents a circle with its centre at ________________.

[(2,0), (3,0), $\left(3, \frac{\pi}{2}\right)$, $(3, \pi)$]

13. Tangent plane of the surface $f(x, y, z) = x^2 + y^2 + z = 0$ at (0,0,0) is ________________.

[$x + y + z = 0$, $2x + 2y + 1 = 0$, $x + y = 0$, $z = 0$]

14. If $x^3 + z^2 + ye^{xz} + z\cos y = 0$, then $\frac{\partial z}{\partial y}$ at $(0,0,0) =$ ________________.

[-1, 0, 1, undefined]

15. $\Gamma\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) =$ ________________ where $\Gamma(n)$ denotes Gamma function of n.

[$\frac{2}{9}$ $\frac{2\pi}{\sqrt{3}}$ $\frac{\sqrt{3}\pi}{2}$, $2\sqrt{3}\,\pi$]

16. $\int_0^1 \int_0^1 \int_0^1 xyz\, dxdydz =$ ________________.

[0, $\frac{1}{8}$, $\frac{1}{3}$, 1]

17. The plane containing unit tangent vector and binormal vector is called ________________.

[normal plane, osculating plane, rectifying plane, principal plane]

18. The divergence of the vector field $\vec{F}(x, y) = 2x\vec{\imath} - y\vec{\jmath}$ is________________.

[0, 1, $y - 2x$, $2x + y$]

19. The primitive period of the function $f(x) = \sin\frac{x}{2}$ is ________________.

[$\frac{\pi}{2}$, π, 2π, 4π]

20. If a function $f(x)$ is odd then, $\int_{-a}^{a} f(x))dx =$________________.

[0, $2a$, $2\int_0^a f(x))dx$, $2\int_{-a}^{a} f(x))dx$]

KATHMANDU UNIVERSITY
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AUG 2 0 2019

Level : B.E./B.Sc./B.Tech.
Year : I
Time : 2 hrs. 30 mins.

Course : MATH 104
Semester: II
F.M. : 55

SECTION "C"
[4Q × 7 = 28 marks]

1. Define divergence and flux of vector field $\vec{F}(x, y)$ across a smooth simple closed curve C. State Green's Theorem in normal form. Verify this form of the theorem for the vector field $\vec{F} = (x-y)\vec{\imath} + x\vec{\jmath}$ and the region R bounded by the unit circle $x^2 + y^2 = 1$. [2+1+4]

OR

Define potential function. Write the partial derivative criteria for a vector field $\vec{F} = M\vec{\imath} + N\vec{\jmath} + P\vec{k}$ to be conservative field. Show that $\vec{F} = (e^x \cos y + yz)\vec{\imath} + (xz - e^x \sin y)\vec{\jmath} + (xy + z)\vec{k}$ is conservative over its natural domain and find its potential function. [1+1+5]

2. Check the symmetry and sketch the graph of the polar curve $r = 2\cos 2\theta$. Find the area of the region shared by the circle $r = 2$ and a cardiod $r = 2 - 2\cos\theta$. [4+3]

3. What is TNB frame? Derive the formula $\tau = -\frac{d\vec{B}}{ds}.\overrightarrow{N}$ for torsion τ. Find $\vec{T}, \vec{N}, \kappa$ for the space curve $\vec{r}(t) = (\cos t + t\sin t)\vec{\imath} + (\sin t - t\cos t)\vec{\jmath} + 3\vec{k}$. [1+3+3]

4. Sketch the region of integration for the integral $\int_0^2 \int_{x^2}^{2x} (4x+2)\,dy\,dx$, write an equivalent integral with the order of integration reversed and evaluate that integral. Find the average value of $F(x, y, z) = xyz$ throughout the cubical region D bounded by the coordinate planes and the planes $x = 2, y = 2, z = 2$. [4+3]

SECTION "D"
[9Q × 3 = 27 marks]

5. Show that $\Gamma(n+1) = n\Gamma(n)$, $\Gamma(n)$ represents the Gamma function.

6. Find the Fourier series expansion of the function $f(x) = \begin{cases} 0 & -\pi < x < -\frac{\pi}{2} \\ 1 & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < x < \pi \end{cases}$.

7. Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P_0(1, 1, 0)$ in the direction of $\vec{v} = 2\vec{\imath} - 3\vec{\jmath} + 6\vec{k}$.

OR

Use chain rule to evaluate $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ if $w = xy + yz + zx, x = u + v, y = u - v, z = uv$ at $(u, v) = (1, 1)$.

8. Find the point on the curve $\vec{r}(t) = (5\sin t)\,\vec{\imath} + (5\cos t)\vec{\jmath} + 12t\vec{k}$ at a distance 26π units along the curve from the point $(0, 5, 0)$ in the direction of increasing arc length.

9. Find the maximum and minimum values of the function $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.

10. Evaluate $\int_C \vec{F}.\vec{T}ds$ for the vector field $\vec{F} = x^2\vec{\imath} - y\vec{\jmath}$ along the curve $x = y^2$ from $(0,0)$ to $(4,2)$.

OR

Find the work done by the force field $\vec{F} = (y - x^2)\vec{\imath} + (z - y^2)\vec{\jmath} + (z - x^2)\vec{k}$ along the line segment joining $(0,0,0)$ and $(1,1,1)$.

11. Evaluate the integral: $\int_0^1 \int_0^{3-3x} \int_0^{3-3x-y} dz\, dy\, dx$.

12. Define $f(0,0)$ in such a way that the function $f(x,y) = xy\frac{x^2-y^2}{x^2+y^2}$ becomes continuous at $(0,0)$.

13. Find the area of surface generated by revolving the right-hand loop of the Lemniscate $r^2 = \cos 2\theta$ about y-axis.