Mark Scored:

KATHMANDU UNIVERSITY
End Semester Examination
August, 2018

Level : B. E./B.Sc./B. Tech. | Course : MATH 104
Year : I | Semester : II
Exam Roll No. : | Time: 30 mins. | F. M. : 20
Registration No.: | Date AUG 17 2018

SECTION "A"

[10 Q ×1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The gradient of the field f(x, y, z) = xyz^2 at point (1, 0, 1) is ....

2. The equivalent Cartesian equation of $r - cos\theta = sin\theta$ is ....

3. Range of the function $f(x, y, z) = \frac{1}{x^2 + y^2 + z^2}$ ....

4. $\left(\frac{\partial f}{\partial x}\right)_z$ = ..., if $f(x, y, z) = x^2 + y^2 + z^2,\ z = x - y$.

5. The double integral $\iint_R rdrd\theta$ over the region **R** gives the ....of the closed and bounded region **R** in polar form.

6. The equation of directrix is .... for parabola $r = \frac{25}{10+10\cos\theta}$.

7. $\int_0^2 \int_0^2 \int_0^2 dzdxdy$ = ...

8. If n is a positive integer, then $\Gamma(n + 1)$ = ..., where the symbol has its usual meaning.

9. The limit of $\lim_{(x,y)\to(0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$ is ....

10. The divergence of the field $\vec{F}(x, y) = xy\hat{i} + y^2\hat{j}$ is ....

SECTION "B"

[10 Q × 1 = 10 marks]

Fill in the blank space(s), DO NOT TICK, by selecting the most appropriate answers from among the given ones.

11. The primitive period of sin $5x$ is..............................

$\left[2\pi; \quad \pi; \quad \frac{\pi}{5}; \quad \frac{2\pi}{5}\right]$

12. The curve $r = 5 + 5\cos\theta$ is symmetric about ....

$[x - axis;$ $y - axis;$ $origin;$ $line\ y = x]$

13. β(1, 2) =......................, where the symbol has its usual meaning.

$\left[1; \quad \frac{1}{2}; \quad \frac{1}{4}; \quad \frac{1}{5}\right]$

14. A vector field $\vec{F}(x, y) = y\hat{i} + x\hat{j}$ is conservative if and only if there exists a scalar potential function $f(x, y) =$ ....

$[x + y;$ $x - y;$ $xy;$ $2xy]$

15. The conic section $= \frac{ke}{1+ecos\theta}$, represents a hyperbola if e is greater than ....

$\left[\frac{1}{2}; \quad \frac{1}{4}; \quad 1; \quad 2\right]$

16. The curvature of a circle of radius a is ....

$\left[\frac{1}{2a}; \quad \frac{1}{a}; \quad a; \quad 2a\right]$

17. If $f(x, y, z) = x^2 + y^2 + z^2$ with $x = \sin t$, $y = \cos t$, $z = 5t$, then $\frac{df}{dt}$ at t = 2 is ....

[0; 100; 200; 300]

18. The plane containing unit tangent vector and binormal vector is called a(n).....................

$[principal\ plane;$ $rectifying\ plane;$ $osculating\ plane;$ $normal\ plane]$

19. The polar equation $r = 3 + 4cos\theta$ is the ....

$[limacons;$ $cardioid;$ $lemniscate;$ $four\ leaved\ rose]$

20. If f(x) is odd function, then $200\int_{-a}^{a} f(x) =$......................

$\left[0; \quad 2\int_{0}^{a} f(x); \quad \int_{a}^{-a} f(x); \quad 2\int_{-a}^{a} f(x)\right]$

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Level : B.E./B. Sc./B. Tech
Year : I
Time : 2 hrs. 30 mins.

Course : MATH 104
Semester : II
F. M. : 55

SECTION "C"
[4 Q × 7 = 28 marks]

1. Define the local extreme value and a saddle point of a function of two variables? Find the absolute maxima and minima of the function $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the region given by a closed triangular plate bounded by the lines $x = 0$, $y = 2$, $y = 2x$ in the first quadrant. [1+1+ 5]

2. When a vector field becomes a conservative vector field? How can we relate a potential function with a conservative field? Show that $\vec{F} = (e^x \cos y + yz)\hat{i} + (xz - e^x \sin y)\hat{j} + (xy + z)\hat{k}$ is conservative and find a potential function for $\vec{F}$. [1+1+2+3]

OR

Define a vector field. State the normal and tangential form of Green's theorem. Verify the tangential form of Green's theorem for the field $\vec{F}(x,y) = (x - y)\hat{i} + x\hat{j}$ and the region R bounded by the unit circle C: $\vec{r}(t) = (\cos t)\hat{i} + (\sin t)\hat{j}$, $0 \le t \le 2\pi$. [1+2+4]

3. Define Beta and Gamma Functions. Prove that:

$$\beta(m,n) = \frac{n-1}{m+n-1}\beta(m, n-1) = \frac{m-1}{m+n-1}\beta(m-1, n).$$ [1+1+ 5]

4. Define a unit tangent vector and a binormal vector. Find $\vec{N}, \vec{B}, \tau$ and κ of $\vec{r}(t) = (\cos t)\vec{i} + (\sin t)\vec{j} - \vec{k}$ at $t = \pi/4$, where the symbols have their usual meanings. [2+5]

SECTION "D"
[9 Q × 3 = 27 marks]

5. Find the average value of $F(x, y, z) = x + y + z$ over the cube in the first octant bounded by the coordinates plane and the planes $x = 1$, $y = 2$, and $z = 3$.

6. Sketch the graph and then find the area of region $r = 3 + 3\cos\theta$.

7. Define first partial derivative of the function $f(x, y)$ with respect to y. Draw a tree diagram and write a chain rule formula for $\frac{\partial w}{\partial r}$ for $w = f(x, y)$, $x = g(r, s)$ and $y = h(r, s)$.

8. Find the derivative of the function $f(x, y) = x^2 + y^2$ at P_o (1, 1) in the direction of unit vector $\vec{u} = 2\hat{i} + 2\hat{j}$.

9. Find the spherical and cylindrical coordinates of the point whose Cartesian coordinate is (0, 1 , 0).

10. Using double integral find the area of the region **R** bounded by $y = 2x^2$ and $y^2 = 4x$.

OR

Evaluate the integral $\int_0^1 \int_0^{2-x} \int_0^{2-x-y} dzdydx$.

11. Define line integral. Find the line integral of $f(x, y, z) = \frac{\sqrt{3}}{(x^2 + y^2 + z^2)}$ over the curve $\vec{r}(t) = t\hat{i} + t\hat{j} + t\hat{k}, 1 \le t < \infty$

12. Find the Fourier series of the function: $f(x) = \begin{cases} 0, & for\ -\pi < x < -\frac{\pi}{2} \\ a, & for\ -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0, & for\ \frac{\pi}{2} < x < \pi \end{cases}$

13. Verify Divergence theorem for the field $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$.