Marks Scored:

KATHMANDU UNIVERSITY
End Semester Examination
August/September, 2017

Level : B. E./B.Sc./B. Tech.
Year : I

Course : MATH 104
Semester : II

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : SEP 0 8 2017

SECTION "A"
[10Q × 1 = 10 marks]

Fill in the blank space(s) by most appropriate word(s) or symbol(s).

1. Equation of circle passing through the origin centered on negative Y-axis with radius a is ______

2. The equation in cylindrical coordinate system of the equation in spherical coordinate system $\rho = 5cos\phi$ is ______

3. If $\vec{u}$ is a differentiable vector function of t of constant length, then $\vec{u}\cdot\frac{d\vec{u}}{dt} =$ ______

4. The curvature of a circle of radius a is ______

5. A region in the plane is bounded if ______

6. The gradient of a differential function at a point is always ______

7. If $\vec{F}$ is conservative then the work done around any closed loop equals ______

8. The area of the surface $f(x, y, z) = c$ over a closed and bounded plane region R is ______

9. The Fourier coefficient a_n of the periodic function $f(t)$ of period T is given by the Euler's formula ______

10. $\int_1^e \int_1^e \int_1^e \frac{1}{xyz} dx\, dy\, dz =$ ______

SECTION "B"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s), DO NOT TICK, by choosing the most appropriate answer from among the given ones.

11. The polar coordinate for Cartesian coordinate $(\sqrt{3},1)$ is ______
[$(-2, -5\pi/6)$; $(-2, -7\pi/6)$; $(2, -5\pi/6)$; $(2, -7\pi/6)$]

12. The equation $\frac{8}{2-4\sin\theta}$ represents ______
[Circle ; Ellipse; Hyperbola; Parabola]

13. If $f(x,y) = y\sin xy$ then $\frac{\partial f}{\partial y} =$ ______________________
[$x^2 \cos xy + \sin xy$; $xy \cos xy + \sin xy$; $y^2 \cos xy + \sin xy$; $xy \cos xy$]

14. Unit tangent vector of a curve $\vec{r}(t) = cost\,\vec{\imath} + \text{sint}\,\vec{\jmath}$ is ______________________
[$\vec{v}$; $sint\,\vec{\imath} - \text{cost}\,\vec{\jmath}$; $-sint\,\vec{\imath} - \text{cost}\,\vec{\jmath}$; $cost\,\vec{\imath} + \text{sint}\,\vec{\jmath}$]

15. The integral $\iint y\delta(x,y)dA$, where δ is the density represents ______________________
[Mass; Area; Moment; Volume]

16. The work done by the conservative field $\vec{F} = yz\vec{\imath} + xz\vec{\jmath} + xy\vec{k}$ along any smooth curve C joining the point $(-1,3,9)$ to $(1,6,-4)$ is ______________________
[3; 4; -3; -4]

17. The divergence of $\vec{F} = 2xz\vec{\imath} - xy\vec{\jmath} - \vec{k}$ is given by the expression ______________________
[$2z + x - 1$; $2z - x + 1$; $2z - x - 1$; $2z + x + 1$]

18. $\int_0^\infty e^{-x^2} dx =$ ______________________
[$\frac{\sqrt{\pi}}{2}$; $\frac{\pi}{2}$; $\frac{1}{\sqrt{\pi}}$; $\frac{2}{\sqrt{\pi}}$]

19. The value of $\beta\left(\frac{1}{3},\frac{2}{3}\right) =$ ______________________
[$\frac{\sqrt{3}}{2}\pi$; $\frac{2}{\sqrt{3}}\pi$; $\frac{3}{\sqrt{2}}\pi$; $\frac{\sqrt{2}}{3}\pi$]

20. ______________________ is the primitive period of $f(x) = \sin 4x$.
[$\frac{\pi}{2}$; π; 2π; 4π]

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Level : B.E./B.Sc./B. Tech.	Course : MATH 104
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Time : 2hrs. 30 mins.	F. M. : 55

SECTION "C"
[4 Q. × 7 = 28 marks]

1. Graph the lemniscate $r^2 = 6\cos 2\theta$. Find the area inside the lemniscates $r^2 = 6\cos 2\theta$ and outside the circle $r = \sqrt{3}$. [3+4]

2. Define directional derivative and state its properties. Using this definition, find the derivative of $f(x,y) = x^2 + xy$ at $P_0(1,2)$ in the direction of unit vector $\vec{u} = \frac{1}{\sqrt{2}}\vec{\imath} + \frac{1}{\sqrt{2}}\vec{\jmath}$. Find the directions in which $f(x,y) = \frac{x^2}{2} + \frac{y^2}{2}$ increases and decreases most rapidly. [1+1+3+2]

3. State Fubini's theorem (stronger form). Find the volume of the solid whose base is the region in the xy – plane that is bounded by the parabola $y = 4 - x^2$ and the line $y = 3x$, while the top of the solid is bounded by the plane $z = x + 4$. [2+5]

OR

State Fubini's theorem (stronger form). Sketch the region of integration and then evaluate the integral $\int_0^{2\sqrt{ln3}} \int_{y/2}^{\sqrt{ln3}} e^{x^2}\, dx\, dy$. [2+5]

4. State Stoke's theorem. Find the circulation of the field $\vec{F} = (x^2 - y)\vec{\imath} + 4z\vec{\jmath} + x^2\vec{k}$ around the curve C in which the plane $z = 2$ meets the cone $z = \sqrt{x^2 + y^2}$, counterclockwise as viewed from above. [2+5]

SECTION "D"
[9 Q. × 3 = 27 marks]

5. Show that the function $f(x,y) = \frac{x^4}{x^4+y^2}$ have no limit as $(x,y) \to (0,0)$.

6. Define Gamma function. Prove that $\Gamma(n+1) = n\Gamma(n)$.

OR

Prove that $B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

7. Find the length of spiral $r = e^{\theta}/\sqrt{2}$ from $\theta = 0$ to $\theta = \pi$.

8. Solve initial value problem: $\frac{d\vec{r}}{dt} = (t^3 + 4t)\vec{\imath} + t\vec{\jmath} + 2t^2\,\vec{k}, \vec{r}(0) = \vec{\imath} + \vec{\jmath}$.

9. Find $\vec{T}, \vec{N}$ and κ for the curve $\vec{r}(t) = \frac{t^3}{3}\vec{\imath} + \frac{t^2}{2}\vec{\jmath},\ t > 0$ where the symbols have their usual meanings.

10. Find the points on the curve $xy^2 = 54$ nearest the origin.

11. Evaluate: $\int_{-1}^{1}\int_{0}^{2\pi}\int_{0}^{1+cos\theta} 4r\, drd\theta dz.$

12. Calculate the outward flux for the field $\vec{F} = (x^2 + 4y)\vec{\imath} + (x + y^2)\vec{\jmath}$ across the square bounded by the lines $x = 0, x = 1, y = 0, y = 1.$

OR

Evaluate $\int_C \vec{F}.\vec{T}\, ds$ for the vector field $\vec{F} = x^2\vec{\imath} - y\vec{\jmath}$ along the curve $x = y^2$ from (4,2) to (1,-1).

13. Find the Fourier series of the function $f(x) = \begin{cases} 0 & if\ -\pi < x < 0 \\ x & if\quad 0 < x < \pi \end{cases}$.