KATHMANDU UNIVERSITY
End Semester Examination [C]
April/May 2023

Marks Scored:

Level : B.E./B.Sc./B.Tech.
Year : I
Course : MATH 104
Semester : II

Exam Roll No.: Time: 30 mins. F. M. : 20

Registration No.: Date : 26 Apr 2023

SECTION "A"
[10Q. × 1 = 10 marks]

Fill in the blank space (s) by most appropriate word (s) or symbol (s).

1. In spherical coordinates, $z = \sqrt{x^2 + y^2}$ is written as $\phi =$ _____.

2. The Cartesian equation for the polar equation $r \sin\theta = \ln r + \ln\cos\theta$ is _____.

3. Direction in which the function $f(x, y, z) = x + y + z$ increases most rapidly at $(0, 0, 0)$ is _____.

4. Find $f_{xy}$ if $f(x, y) = x + \frac{y}{x}$ _____.

5. The equivalent polar form of the integral $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy\, dx$ is _____.

6. $\beta(2, 3) =$ _____, where the symbol has its usual meaning.

7. Speed of a particle whose motion in space is given by $\vec{r}(t) = 3\cos t\ \vec{\imath} + 4\sin t\,\vec{\jmath} + \vec{k}$ is _____.

8. The plane containing unit tangent vector, $\vec{T}$ and binormal vector, $\vec{B}$ is called _____ plane.

9. Curl of a vector field, $\vec{F} = z^2\,\vec{\imath} + y^2\vec{\jmath} + x^2\,\vec{k}$ is _____.

10. If $f(x)$ is an odd function, then $\int_{-\pi}^{\pi} f(x)\cos x\ dx =$ _____.

SECTION "B"
[10Q. × 1 = 10 marks]

Fill in the blank space (s) by choosing the most appropriate answer from among the given ones. **Do not tick** the answers.

11. Which polar coordinate labels the same as $\left(2, \frac{\pi}{3}\right)$? _____.
[$\left(-2, -\frac{\pi}{3}\right)$, $\left(2, \frac{2\pi}{3}\right)$, $\left(2, \frac{4\pi}{3}\right)$, $\left(-2, \frac{4\pi}{3}\right)$]

12. The equation of directrix of the conic $r = \frac{10}{10 - 10\sin\theta}$ is _____.
[$x = 1$, $x = -1$, $y = 1$, $y = -1$]

13. Hessian of the function $f(x,y) = x^2 + y^2$ at $(1,1)$ is _____.
[0, 1, $2x + 2y$, 4]

14. If a function $z = f(x,y)$ is differentiable function of $x$, $y$, and $x$, $y$ are functions of $t$, then z is differentiable function of $t$ and _____.
[$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$, $\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$,
$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$, $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$]

15. The integral value of $F(x,y,z) = xyz$ over the cube bounded by the coordinate planes and the planes $x = 1$, $y = 1$, $z = 1$ is _____.
[$\frac{1}{3}$, 1, 2, 3]

16. The value of $\frac{\Gamma\left(\frac{9}{2}\right)}{\Gamma\left(\frac{7}{2}\right)}$ is _____.
[$\frac{9}{7}$, $\frac{9}{2}$, $\frac{7}{2}$, $\frac{1}{2}\sqrt{\pi}$]

17. Radius of the curvature of the curve $r = 2\cos t\,\vec{i} + 2\sin t\,\vec{j}$ is _____.
[$\frac{1}{4}$, $\frac{1}{2}$, 2, 4]

18. For any function $f(x,y,z)$ whose partial derivatives are continuous, $\nabla \times \nabla f =$ _____.
[$-1$, 0, $\nabla f$, $f$]

19. If a field $\vec{F} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$ is conservative, then $\nabla f =$ _____ where $f$ is the potential function of the vector field $\vec{F}$.
[$x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$, $\vec{i} + \vec{j} + \vec{k}$, $-x\,\vec{i} - y\,\vec{j} - z\,\vec{k}$ $xyz$]

20. If $f(x)$ and $g(x)$ each have periods $p$, then the period of the function $h(x) = af(x) + bg(x)$ ($a, b$ are constants) is _____.
[$\pi$, $2\pi$, $p$, $2p$]