KATHMANDU UNIVERSITY
End Semester Examination
September 2024

Marks Scored:

Level : B.Sc.
Year : I

Course : MATH 103
Semester : II

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : 05 SEP 2024

SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. A finite dimensional real inner product space is ____________ space.

2. A set of vectors in the vector space containing the zero vector is __________

3. For the real valued function $h(x) = x^2 - 4$, the pre-image of 5 is ____________

4. The square of length of vector $\boldsymbol{a} = (-1, -2, 3, -4, -5)$ is ________________

5. The ____________ elements of a skew-Hermitian matrix are imaginary.

6. The dimension of the finite dim kernel of a linear mapping on vector spaces is __________

7. The direction of the vector projection of **u** along **v** is the same as that of **v** if ________

8. The span $[\mathbf{v}_1, \mathbf{v}_2]$ of two non-collinear vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ represents ________ through these vectors.

9. The solution of a linear system AX = B is unique if the matrix A is square & _________

10. For a subspace U of a vector space V and for $v \in V$, the set $\{v\} + U$ is a linear _________

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The inner product of two vectors $\mathbf{x} = (2+i, i, 2i)$ and $\mathbf{y} = (2 - i, i, i)$ is _________
[-1 ; 0; 1; i]

12. The cosine of the angle between $\vec{A} = 10\hat{i} + 11\hat{j} - 2\hat{k}$ and $\vec{B} = 3\hat{j} + 4\hat{k}$ is _______

[- 1/3; - 1/2; 1/3; 1/2]

13. The product of eigen values of a matrix $\begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$ is _______

[- 4; -1; 1; 4]

14. If U and V are vector spaces with dim U = 2 and dim V = 3, then the dimension of the set ***L***(U, V) of all linear maps from U to V, is _______

[1; 3; 5; 6]

15. For real mappings $f(x) = x^2+x+1$ and $g(x) = 2x - 3$, $(gof)(-1)$ is ________

[- 2; -1; 1; 2]

16. The rank of the matrix $\begin{bmatrix} 3 & 2 & 3 & 1 \\ 4 & 3 & 5 & 2 \\ 2 & 1 & 1 & 0 \end{bmatrix}$ is ______

[1; 2; 3; 4]

17. For the vectors **u** and **v** are orthonormal, the value of $\|u + v\|$ is _______

[0; 1; $\sqrt{2}$; 2]

18. A square matrix A is said to be ________ if $A^T = A$.

[symmetric ; asymmetric ; idempotent ; skew-symmetric]

19. The kernel of the linear transformation T: $V_3 \rightarrow V_3$ defined by $T(x_1, x_2, x_3) = (x_1 - x_2, x_1+x_3, x_2+x_3)$ is ______

[(1, -1, -1); {(1, -1, -1)}; (1, 1, -1); {(1, 1, -1)}]

20. The scalar projection of **u** = (2, -3, 1) on **v** = (1, 1, 1) is ____________

[-1; 0; 1/2; 1]

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SECTION "C"
[3Q × 7 = 21 marks]

1. Define a field and the vector space over a field. Also, show that the set of all complex numbers form a vector space over the real field under the usual addition and scalar multiplication. [2+2+3]

2. For linear maps T : U →V and S: V→W on the vector spaces U, V, W over the same field of scalar, define the composition of S and T. Also, prove that the composition ST is linear and if ST is non-singular, then prove that T is one-one, symbols have their usual meanings. [2+3+2]

OR

What is meant by a basis on a vector space? Prove that any set of n linearly independent vectors in n-dimensional vector space V_n is a basis. Also, verify that the set of vectors {(1, 1, -1), (1, -1, 1), (-1, 1, 1)} forms a basis for $\Re^3$. [2+3+2]

3. Define a characteristic equation associated to a square matrix and its eigen values. Also, find the eigen space for the matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$. [2+1+4]

SECTION "D"
[6Q × 4 = 24 marks]

4. If T is a one-one linear map from vector space U to another vector space V and $u_1, u_2, \ldots, u_n$ are **LI** vectors of U, then show that $T(u_1), T(u_2), \ldots, T(u_n)$ are also **LI**, symbols have their usual meanings.

5. If U and W are two subspaces of a vector space V, show that U +W is a subspace and also prove that $U + W = [U \cup W]$, symbols have their usual meanings.

OR

Find the matrix associated to the linear transformation T: $\Re^3 \to \Re^2$ defined by
$T(x, y, z) = (2x - y + 3z, x - 2y + 4z)$ relative to the basis
$\boldsymbol{B}_1$ = {(1, 1, 1), (1, 1, 0), (1, 0, 1)} and
$\boldsymbol{B}_2$ = {(1, 2), (2, 3)} of $\Re^3$ and $\Re^2$ respectively.

P.T.O.

6. Show that the mapping T : $V_3 \to V_2$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2, x_1 - x_3)$ is linear. Also, find its range and nullity.

7. Use the Gram-Schmidt process of vectors to orthonormalize the set of linearly independent vectors {(1, 1, 1), (0, 1, 2), (5, -1, 2)} of V_3.

8. If A and B are matrices of order 3 over the set of complex numbers, then verify that (AB)* = B*A*, where A* denotes the tranjugate of A.

9. Use matrix method to solve the system the system of linear equations:
$2x - y + z = -2$, $x - y - 2z = -9$, $x - 2y - z = 9$. Also, verify the answer.

SECTION "E"
[5Q × 2 = 10 marks]

10. Find the vector projection of **u** = (-1, 1, 1) along **v** = (3, -2, 1).

11. For non-empty sets A, B, C, prove that $A\times(B\cap C) = (A\times B)\cap(A\times C)$ symbols have usual meanings.

12. Show that the intersection of two subspaces of a vector space is also a subspace.

13. Also, in the complex vector space V_2^C, show that (1+i, 1- i) belongs to [(1+i, 1), (1, 1- i)].

14. Find the coordinate vector of (-1, 3, 1) of V3 relative to basis set
{(2, 1, 0), (2, 1, 1), (2, 2, 1)}.