Marks scored:

KATHMANDU UNIVERSITY
End Semester Examination [C]
January, 2019

JAN 0 2 2019

Level : B.Sc.
Year : I
Exam Roll No. :

Registration No.:

Course : MATH 103
Semester: II
F. M. : 20
Time: 30 mins.

Date :

SECTION "A"
[10 Q × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbols(s).

1. The .... of the absolute values of the eigen value of square matrix is its spectral radius.

2. The set $\{v_1, v_2, v_3\}$ is linearly dependent if v_1, v_2, v_3 are ....

3. For vectors $\mathbf{a}$ and $\boldsymbol{x}$ in $\Re^n$, the equation $\mathbf{a} \cdot \boldsymbol{x} = 0$ is .... through origin.

4. The length of vector $\mathbf{b} = (1, -2, 3, -4)$ is ....

5. The diagonal elements of a skew-Hermitian matrix are ....

6. A vector space of dimension α over a field $\Re$ is .... to $\Re^\alpha$.

7. The direction of the vector projection of $\mathbf{u}$ along $\mathbf{v}$ is the same as that of $\mathbf{v}$ if ....

8. A relation from A to B is a subset of ....

9. A finite dimensional real inner product space is .... space.

10. For a subspace U of a vector space V and a vectors v of V, $v + U$ is .... of U by v.

SECTION "B"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answer from among the given ones.

11. The cosine of the angle between $\vec{A} = 10\hat{i} + 11\hat{j} - 2\hat{k}$ and $\vec{B} = 3\hat{j} + 4\hat{k}$ is...........
[- 1/3; - 1/2; 1/3; 1/3]

12. The inner product of two vectors $\alpha = (2, i, 1)$ and $\beta = (2+i, i, 2)$ is ....
[-7 - 2i; 7 – 2i; -7 + 2i; 7 + 2i]

13. The product of eigen values of a matrix $\begin{bmatrix} 2 & 5 \\ 4 & 1 \end{bmatrix}$ is ....

[- 18; - 12; 12; 18]

14. For vector spaces U and V with dim U = 7 and dim V = 5, then the dimension of the set L(U, V) of all linear maps from U to V, is ....

[5; 7; 25; 35]

15. If $A = \begin{bmatrix} 1+i & -i & 2 \\ 4+i & 1-i & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1-i \\ 1 & 2 \\ 1-i & i \end{bmatrix}$, then AB is ....

[$\begin{bmatrix} 2-3i & 2 \\ 1-i & 7-5i \end{bmatrix}$; $\begin{bmatrix} 2+3i & 2 \\ 1-i & 7-5i \end{bmatrix}$; $\begin{bmatrix} 2-3i & 2 \\ 1+i & 7-5i \end{bmatrix}$; $\begin{bmatrix} 2-3i & 2 \\ 1-i & 7+5i \end{bmatrix}$]

16. The rank of the matrix $\begin{bmatrix} 3 & 2 & 3 & 1 \\ 4 & 3 & 5 & 2 \\ 2 & 1 & 1 & 0 \end{bmatrix}$ is ....

[1; 2; 3; 4]

17. The norm of P(2, -1, 2) is ......

[2; 3; 4; 5]

18. A square matrix A is said to be .... if $A^2 = A$.

[symmetric ; asymmetric ; idempotent ; singular]

19. The kernel of the linear transformation T defined on $\mathfrak{R}^2$ by $T(x_1, x_2) = (0, x_1 + x_2)$ is ......

[(1, -1); {(1, -1)}; {(-1, 1)}; (1, 1)}]

20. The value of the determinant of a square matrix $\begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$ is ....

[- 4; - 2; 2; 4]

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Level : B.Sc.
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Time : 2 hrs. 30 mins.

Course : MATH 103
Semester: II
F. M. : 55

SECTION "C"

[3 Q. × 7 = 21 marks]

1. Define a field and a vector space over a field. Also, prove that the set of all matrices of order 3 over a real field forms a vector space under the operations of matrix addition and scalar multiplication. [1+2+4]

2. What is meant by the composition of linear maps on vector spaces over the same field of scalars? If T_1, T_2 are linear maps from U to V and S_1 is a linear map from V to W, where U, V, W are vector spaces over the same fields of scalars, then prove that (i) T_1+T_2 is a linear map from U to V, and (ii) $S_1(T_1+T_2) = S_1T_1 + S_1T_2$. [3+2+2]

OR

What is meant by a basis on a vector space? Prove that any set of n linearly independent vectors in n-dimensional vector space V_n is a basis. Also, verify that the set of vectors $\{(1, 2, 1), (2, 1, 1), (1, 1, 2)\}$ forms a basis for $\Re^3$. [3+2+2]

3. Define a characteristic equation associated to a matrix and its eigen values. Also, find the eigen values and associated vector(s) of the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$. [2+1+4]

SECTION "D"

[6 Q. × 4 = 24 marks]

4. If T is a one-one linear map from vector space U to another vector space V and $u_1, u_2, \ldots, u_n$ are **LI** vectors of U, then show that $T(u_1), T(u_2), \ldots, T(u_n)$ are also **LI**, symbols have their usual meanings.

5. Consider the linear map $T : V_4 \to V_3$ defined by $T(e_1) = (1, 1, 1)$, $T(e_2) = (1, -1, 1)$, $T(e_3) = (1, 0, 0)$, $T(e_4) = (1, 0, 1)$. Then, verify that $r(T) + n(T) = \dim V_4$, where the symbols have their usual meanings.

OR

Find the matrix representation of the linear transformation $T: \Re^2 \to \Re^2$ defined by $T(x, y) = (x, x - 2y)$ relative to the basis $\{(1, 0), (-1, 1)\}$.

6. Show that the span of a non-empty subset S of a vector space V is the smallest subspace of V containing S.

7. Apply Gram-Schmidt process to orthonormalize the set of linearly independent vectors $\{(1, 0, 1, 1), (-1, 0, -1, 1), (0, -1, 1, 1)\}$ of V_4.

8. If A and B are matrices of order 3 over the set of complex numbers, then verify that $(AB)^* = B^*A^*$, where A^* denotes the tranjugate of A.

9. Use matrix method to solve the system the system of linear equations: $x + y + z = 3$, $2x + 2y + z = 5$, $3x + 2y + 2z = 7$.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Find the vector projection of $\vec{B} = 2\hat{i} + 2\hat{j} + \hat{k}$ on $\vec{A} = \hat{i} + 2\hat{j} - \hat{k}$.

11. Show that the mapping T defined on real space $\Re^2$ defined by $T(x, y) = (2x + y, (x + 3y)$ is linear.

12. Prove that the union of two subspaces of a vector space is also a vector space.

13. Show that the eigen value of a matrix and its transpose are the same.

14. If $T : U \rightarrow V$ is a linear mapping where U, V are vector spaces, show that T is one-one if and only if its kernel is the zero subspace of U.