KATHMANDU UNIVERSITY
End Semester Examination
May/June 2022

Mark Scored:

Level : B.Sc.
Year : I
Exam Roll No. :
Registration No.:

Time: 30 mins.

Course : MATH 103
Semester: II
F.M. : 20
Date :

SECTION "A"

[10Q × 1 = 10 marks]

Fill in the blank space (s) by most appropriate word (s) or symbol (s).

1. A system of linear equations is inconsistent if it has ________________ solution.
2. A square matrix is ______________ if its column vectors are linearly independent.
3. The n-vector with only one component is ____________________.
4. The length of vector **a** = (1, 2, -3, - 4, 5) is ____________________.
5. The diagonal elements of a Hermitian matrix are ____________________.
6. The nullity of a matrix is the ____________________ of its kernel.
7. The direction of the vector projection of **u** along **v** is the same as that of **v** if ____________________.
8. The span of two non-collinear vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ is ____________________ through these vectors.
9. A finite dimensional complex inner product space is ____________________ space.
10. If U is the line $y = x$ through the origin in V_2 and a point $v = (1, 0)$, then the translate $v + U$ of U by v is ____________________ through the point (1, 0).

SECTION "B"

[10Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The cosine of the angle between vectors **u** = (10, 11, -2) and **v** = (0, 3, 4) is…………
[- 1/3; - 1/2; 1/3; 1/3]
12. The inner product of vector **u** = (2+ i, 4 – 3i) with itself is ………..
[25; 30; 35; 40]

13. The product of eigen values of a matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is ....

[-1; 0; 1; 2]

14. For vector spaces U and V with dim U = 2 and dim V = 3, the dimension of the set $\boldsymbol{L}$(U, V) of all linear mappings from U to V, is ....

[1; 3; 5; 6]

15. The scalar projection of u = (1, 3, 5) on v = (-3, 1, 1) is $\alpha/\sqrt{11}$ where α is ....

[1; 3; 5; 7]

16. The rank of the matrix $\begin{bmatrix} 3 & 2 & 3 & 1 \\ 4 & 3 & 5 & 2 \\ 2 & 1 & 1 & 0 \end{bmatrix}$ is ....

[1; 2; 3; 4]

17. If the vectors **u** and **v** are orthonormal, then $\|u+v\|^2$ is ....

[0; 1; $\sqrt{2}$; 2]

18. A square matrix A is said to be .... if $A^T = A$.
[symmetric ; asymmetric ; idempotent ; singular]

19. The kernel of the linear transformation S : $V_2 \rightarrow V_2$ defined by $S(x_1, x_2) = (x_1 + x_2, 0)$ is ....
[{(1, -1)}; (1, -1); (-1, 1); {(-1, 1)}]

20. The value of the determinant of a square matrix $\begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$ is ....

[- 4; - 2; 2; 4]

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Level : B.Sc.
Year : I
Time : 2 hrs. 30 mins.

Course : MATH 103
Semester: II
F.M. : 55

SECTION "C"

[3Q × 7 = 21 marks]

1. Define a vector sub-space over a field and its span. Also, show that the span of a non-empty subset S of a vector space V is the smallest subspace of V containing S. [2+1+4]

2. What is meant by the composition of linear maps on vector spaces over the same field of scalars? If T_1, T_2 are linear maps from U to V and S_1 is a linear map from V to W, where U, V, W are vector spaces over the same fields of scalars, then prove that (i) T_1+T_2 is a linear map from U to V, and (ii) $S_1(T_1+T_2) = S_1T_1 + S_1T_2$. [3+2+2]

OR

What is meant by a basis on a vector space? Prove that any set of n linearly independent vectors in n-dimensional vector space V_n is a basis. Also, verify that the set of vectors {(1, 2, 8), (0, 1, 5), (4, -1, 0)} forms a basis for $\mathfrak{R}^3$. [3+2+2]

3. Define a characteristic equation associated to a matrix and its eigen values. Also, find the eigen values and associated vector(s) of the matrix $A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$. [2+1+4]

SECTION "D"

[6Q × 4 = 24 marks]

4. If T is a one-one linear map from vector space U to another vector space V and $u_1, u_2, \ldots, u_n$ are **LI** vectors of U, then show that $T(u_1), T(u_2), \ldots, T(u_n)$ are also **LI**, symbols have their usual meanings.

5. Consider the linear map $T : V_4 \to V_3$ defined by $T(e_1) = (1, 1, 1)$, $T(e_2) = (1, -1, 1)$, $T(e_3) = (1, 0, 0)$, $T(e_4) = (1, 0, 1)$. Then, verify that $\boldsymbol{r}(T) + \boldsymbol{n}(T) = \dim V_4$, where the symbols have their usual meanings.

OR

Find the matrix representation of the linear transformation $T: \mathfrak{R}^2 \to \mathfrak{R}^2$ defined by $T(x, y) = (x, x - 2y)$ relative to the basis {(1, 0), (-1, 1)}.

6. Find the inverse of the real mapping T defined on $\mathfrak{R}^2$ by $T(x, y) = (x - 2y, 2x + y)$ and also verify your answer.

7. Apply Gram-Schmidt process to orthonormalize the set of linearly independent vectors {(1, 0, 1, 1), (-1, 0, -1, 1), (0, -1, 1, 1)} of $\mathbf{V}_4$.

8. Show that the set $\mathbf{V}_4$ of all 4-tuples of real numbers is a vector space with usual addition and scalar multiplication of real numbers.

9. Use matrix method to solve the system the system of linear equations: $2x - 3y + z = -1$, $3x + z = 6$, $x + 2y - 2z = -1$.

SECTION "E"

[$5Q \times 2 = 10$ marks]

10. For non-empty sets A, B, C, prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$, where the symbols have their usual meanings.

11. Show that the set $\{v_1, v_2, v_3\}$ in a vector space is **LI** if v_1, v_2, v_3 are coplanar.

12. Prove that the real mapping $T(x_1, x_2, x_3)$ defined on $\Re^3$ by $(x_1+x_2, x_2+x_3, 0)$ is linear.

13. Show that a linear transformation defined on vector spaces is one-one iff its kernel is the zero subspace of the domain space.

14. If the vector space V has a basis of finite elements then show that every other basis for V has the same elements.