

KATHMANDU UNIVERSITY
End Semester Examination [C]
January, 2018

Mark Secured:

Level : B. Sc.

Year : I

Exam Roll No. :

Time: 30 mins

Course : MATH 103

Semester : II

F. M. : 20

Registration No. :

Date JAN 16 2018

SECTION "A"

[10Q. \times 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbols(s).

1. The set of eigen values of the square matrix is _____
2. If V has a basis element of 10 elements, then every set of 12 vectors is _____
3. Every 2-dimensional subspace of V_3 is a plane through _____
4. The length of vector $\mathbf{u} = (1, -2, 4, 2)$ is _____
5. The diagonal elements of a skew-Hermitian matrix are _____
6. The determinant of $(A - \lambda I)$ in the characteristic equation of the matrix A is _____ in λ .
7. The dimension of the vector space is _____
8. A square matrix is non-singular if its _____ vectors are linearly independent.
9. If $T : U \rightarrow V$ is a linear mapping with $\dim U = \dim V = k$, then the nullity of T is _____
10. If U is a subspace of a vector space V and v a vector of V , then $v + U$ is _____ of U (by v).

SECTION "B"

[10 Q. \times 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answer from among the given ones.

11. The positive value of x such that the vectors $(1, 2, 1)$, $(x, 3, 1)$ and $(2, x, 0)$ are linearly dependent, is _____
[1; 2; 3; 4]
12. The inner product of two vectors $\mathbf{u} = (1, 2, -3)$ and $\mathbf{v} = (0, -1, 2)$ is _____
[- 8; - 4; 4; 8]

13. The sum of eigen values of a matrix $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ is _____
 [- 3; 1; 2; 3]
14. If U and W are vector subspaces of V with $\dim U = 2$, $\dim W = 3$ and $\dim(U+W) = 4$, then the $\dim(U \cap W)$ is _____
 [0; 1; 2; 3]
15. If $A = \begin{bmatrix} 1+i & -i & 2 \\ 4+i & 1-i & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1-i \\ 1 & 2 \\ 1-i & i \end{bmatrix}$, then AB is _____
 [$\begin{bmatrix} 2-3i & 2 \\ 1-i & 7-5i \end{bmatrix}$; $\begin{bmatrix} 2+3i & 2 \\ 1-i & 7-5i \end{bmatrix}$; $\begin{bmatrix} 2-3i & 2 \\ 1+i & 7-5i \end{bmatrix}$; $\begin{bmatrix} 2-3i & 2 \\ 1-i & 7+5i \end{bmatrix}$]
16. If $T : V_2 \rightarrow V_2$ and $S : V_2 \rightarrow V_2$ are two linear mappings defined by $S(x_1, x_2) = (x_1 + x_2, x_1 - x_2)$ and $T(x_1, x_2) = (3x_2, x_1 + x_2)$ then $(S + T)(1, -1)$ is _____
 [(-2, -2); (2, -2); (-2, 2); (2, 2)]
17. For two orthonormal vectors u and v , $\|u + v\|$ is _____
 [0; 1; $\sqrt{2}$; 2]
18. A square matrix A is said to be _____ if $A^2 = A$.
 [symmetric; asymmetric; idempotent; involutory]
19. The kernel of the linear transformation $T : V_3 \rightarrow V_2$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2, x_1 - x_3)$ is _____
 [(1, -1, 1); $\{(1, -1, -1)\}$; (1, 1, -1); $\{(1, 1, -1)\}$]
20. The value of the determinant of a square matrix $\begin{bmatrix} 2 & -3 & 1 \\ 1 & 1 & -1 \\ 2 & -3 & 1 \end{bmatrix}$ is _____
 [- 1; 0; 1; 2]

KATHMANDU UNIVERSITY
End Semester Examination [C]
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Level : B. Sc.
Year : I
Time : 2 hrs. 30 mins.

Course : MATH 103
Semester : II
F. M. : 55

SECTION "C"

[3 Q. × 7 = 21 marks]

1. Define a real field and a vector space over a field. Also, show that the set C of all complex numbers forms a vector space over the set of real numbers with usual addition and scalar multiplication defined on it. [2+2+3]
2. What is meant by the composition of linear maps on vector spaces over the same field of scalars? If T is a linear map from U to V and S is a linear map from V to W , where U, V, W are vector spaces over the same field of scalars, and if ST is non-singular then prove that T is one-one and S is onto. [2+2+3]

OR

Define a basis on a vector space with example. Prove that any set of n linearly independent vectors in n -dimensional vector space V_n forms a basis. Also, show that the set $S = \{(1, 2, 1), (2, 1, 1), (1, 1, 2)\}$ is a basis for V_3 . [3+2+2]

3. Define a characteristic equation associated to a matrix and its eigen values. Also, find the eigen values and spectrum of the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$. [2+1+4]

SECTION "D"

[6 Q. × 4 = 24 marks]

4. If T is a linear map from vector space U to another vector space V and if $[u_1, u_2, \dots, u_n] = U$, then show that $R(T) = [T(u_1), T(u_2), \dots, T(u_n)]$, symbols have their usual meanings.
5. Find the matrix of linear transformation $T : V_2 \rightarrow V_3$ defined by $T(x_1, x_2) = (x_1 + x_2, 2x_1 - x_2, 7x_2)$ relative to standard bases $B_1 = \{(1, 0), (0, 1)\}$ and $B_2 = \{(1, 0, 0), (0, 1, 1), (0, 0, 1)\}$, where the symbols have their usual meanings.

OR

State and prove the Rank-Nullity Theorem of finite dimensional vector space.

6. Consider the vector space $V = C[0, 1]$ and define $f \cdot g = \int_0^1 f(t)g(t)dt$ for f, g in V . Then, verify that it is an inner product space.
7. Apply Gram-Schmidt process to orthonormalize the set of linearly independent vectors $\{(1, 2, 0), (1, 0, -1), (2, 2, 1)\}$ of V_3 .

8. If A and B are matrices of order 3 over the set of complex numbers, then verify that $(AB)^* = B^*A^*$, where A^* denotes the tranjugate of A .
9. Use matrix method to solve the system the system of linear equations: $x + y + z = 3$, $2x + 2y + z = 5$, $3x + 2y + 2z = 7$.

SECTION "E"

[5 Q. \times 2 = 10 marks]

10. Find the vector projection of $\vec{B} = \hat{i} + \hat{j} + \hat{k}$ on $\vec{A} = 5\hat{j} - 3\hat{k}$.
11. Show that the mapping $T : V_2 \rightarrow V_2$ defined by $T(x, y) = (2x + 3y, 3x - 4y)$ is linear.
12. Prove that the sum of any two subspaces of a vector space is also a subspace.
13. Verify that the set $\{1 - x^2, x + 2x^2, 1 + 3x^2\}$ is linearly independent in the vector space P_2 of polynomial over the set of real numbers.
14. For the matrices $A = \begin{bmatrix} 1 & i \\ i & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & i \\ -i & -1 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.