KATHMANDU UNIVERSITY
End Semester Examination
August, 2019

Mark Scored:

Level : B.Sc.
Year : I

Course : MATH 103
Semester: II

Exam Roll No. :

Time: 30 mins.

F.M. : 20

Registration No.:

Date AUG 2 0 2019

SECTION "A"

[10Q × 1 = 10 marks]

Fill in the blank space (s) by most appropriate word (s) or symbol (s).

1. The numbers that constitute a matrix are called ....

2. A system of linear equations is said to be .... if it has one or more columns.

3. The set of vectors containing null vector is ....

4. A vector space of dimension 5 over a field $\Re$ is isomorphic to ....

5. The diagonal elements of a skew-Hermitian matrix are ....

6. The .... of a matrix is the dimension of its kernel.

7. The direction of the vector projection of **u** along **v** is the same as that of **v** if ....

8. The .... of two non-collinear vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ is plane through these vectors.

9. The inverse of the matrix $\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$ is ....

10. The .... of two subspace of a vector space over a field is its subspace.

SECTION "B"

[10Q × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answer from among the given ones.

11. The cosine of the angle between $\vec{P} = 10\hat{i} + 11\hat{j} - 2\hat{k}$ and $\vec{Q} = 3\hat{j} + 4\hat{k}$ is...........
[- 1/3; - 1/2; 1/2; 1/3]

12. The inner product of two vectors **u** = (-1, 1, 2, 4) and **v** = (1, 2, -1, 1) is ....
[- 3; - 1; 1; 3]

13. The product of eigen values of a matrix $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ is ....
[6; 8; 10; 12]

14. The size of the matrix represented by a linear transformation T : $\Re^3 \to \Re^5$ is ....
[3×3; 5×3; 3×5; 5×5]

15. If $C = \begin{bmatrix} 1+i & -i & 2 \\ 4+i & 1-i & 0 \end{bmatrix}$ and $D = \begin{bmatrix} 0 & 1-i \\ 1 & 2 \\ 1-i & i \end{bmatrix}$, then CD is ....
[$\begin{bmatrix} 2-3i & 2 \\ 1+i & 7-5i \end{bmatrix}$; $\begin{bmatrix} 2-3i & 2 \\ 1-i & 7+5i \end{bmatrix}$; $\begin{bmatrix} 2-3i & 2 \\ 1-i & 7-5i \end{bmatrix}$; $\begin{bmatrix} 2+3i & 2 \\ 1-i & 7-5i \end{bmatrix}$]

16. The unit vectors in the direction of $\bar{u} = (1, -1, 1)$ is $v/\sqrt{3}$ where v is ....
[(-1, 1, 1); (1, -1, 1); (1, 1, -1); (1, 1, 1)]

17. The length of the vector (1, 2, 4, - 3) is .... Unit.
[0; 5; $\sqrt{30}$; 6]

18. A square matrix A is said to be .... if $A^2 = A$, an identity matrix.
[symmetric ; involuntary ; idempotent ; singular]

19. The kernel of the linear transformation T : $V_2 \to V_2$ defined by $T(x_1, x_2) = (0, x_1 + x_2)$ is ....
[{(1, -1)}; {(-1, 1)}; {(0, 0)}; {(1, 1)}]

20. The cofactor of 3 in the square matrix $\begin{bmatrix} 1 & -2 & 1 \\ 2 & 3 & -1 \\ 1 & 2 & 2 \end{bmatrix}$ is ....
[- 1; 0; 1; 2]

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AUG 2 0 2019

Level : B.Sc.
Year : I
Time : 2 hrs. 30 mins.

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F.M. : 55

SECTION "C"
[$3Q \times 7 = 21$ marks]

1. Define a field and a vector space over a field. Also, show that the set of all 3-tuples forms a vector space under the usual addition and scalar multiplication defined on it. [1+2+4]

2. Define a linear transformation on vector spaces over a field. Show that a linear transformation is completely determined by its values on the elements of a basis. Also, verify the linearity of mapping $T : V_3 \to V_2$ given by $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + x_3)$. [2+3+2]

OR

What is meant by a basis on a vector space? Prove that any set of n linearly independent vectors in n-dimensional vector space V_n is a basis. Also, verify that the set of vectors $\{(1, 2, 1), (2, 1, 1), (1, 1, 2)\}$ forms a basis for $\Re^3$. [2+3+2]

3. Define an eigen value and associated vector of a matrix. Also, determine the eigen space of the matrix $A = \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$. [2+1+4]

SECTION "D"
[$6Q \times 4 = 24$ marks]

4. If T is a one-one linear map from vector space U to another vector space V and $u_1, u_2, \ldots, u_n$ are **LI** vectors of U, then show that $T(u_1), T(u_2), \ldots, T(u_n)$ are also **LI**, symbols have their usual meanings.

5. If S and T are linear transformations on $\Re^2$ and both S and T are non-singular, then show that (i) ST is non-singular and (ii) $(ST)^{-1} = T^{-1}S^{-1}$.

OR

Find the matrix representation of the linear transformation T: $\Re^2 \to \Re^2$ defined by $T(x, y) = (x, x - 2y)$ relative to the basis $\{(1, 0), (-1, 1)\}$.

6. If T is a linear mapping from vector space U to another vector space V over the same field, then prove that T is one – one if and only if the Kernel of T is zero subspace of U.

7. Apply Gram-Schmidt process to orthonormalize the set of linearly independent vectors $\{(1, 1, 1), (0, 1, 2), (2, 1, 1)\}$ of V_3.

8. If A and B are matrices of order 3 over the set of complex numbers, then verify that $(AB)^* = B^*A^*$, where A^* denotes the tranjugate of A.

9. Use matrix method to solve the system the system of linear equations: $x + y + 2z = 3$, $2x + 2y + 2z = 7$, $3x + 4y + 3z = 2$.

SECTION "E"

[$5Q \times 2 = 10$ marks]

10. Find the vector projection of Q (1, 2, 3) onto P (- 2, 3, 7).

11. Find the inverse of the real mapping $g(x)$ defined by $g(x) = 3x + 1$.

12. If S is a nonempty subset of a vector space V, then show that the span of S is a subspace of V.

13. Show that the eigen value of a matrix and its transpose are the same.

14. In any vector space V, prove that $(-1)\mathbf{u} = \mathbf{u}$, for each $\mathbf{u}$ in V.