Mark Scored:

KATHMANDU UNIVERSITY
End Semester Examination
August, 2018

Level : B.Sc.
Year : I

Course : MATH 103
Semester: **II**

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : AUG 17 2018

SECTION "A"

[10Q × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbols(s).

1. A finite dimensional complex inner product space is .... space.

2. A set of vectors in the vector space containing the zero vector is ....

3. For the real valued function $h(x) = x^2$, the pre-image of -1 is ....

4. The length of vector $\mathbf{b} = (1, 2, -3, -4, 5)$ is ....

5. The diagonal elements of a skew-Hermitian matrix are ....

6. Every real vector space of dimension 5 is .... to V_5.

7. The direction of the vector projection of **u** along **v** is the opposite as that of **v** if ....

8. The span of two non-collinear vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ is .... through these vectors.

9. The determinant of matrix $\begin{bmatrix} 1 & 1 & 3 \\ 2 & 9 & 1 \\ 4 & 11 & 7 \end{bmatrix}$ is ....

10. If U is the line $y = x$ through the origin in V_2 and a point $v = (2, 0)$, then the translate $v + U$ of U by v is .... through the point (1, 0).

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answer from among the given ones.

11. The cosine of the angle between $\vec{A} = 10\hat{i} + 11\hat{j} - 2\hat{k}$ and $\vec{B} = 3\hat{j} + 4\hat{k}$ is...........
[- 1/3; - 1/2; 1/3; 1/3]

12. The inner product of two vectors $\mathbf{x} = (2+i, i, 2)$ and $\mathbf{y} = (2, i, 1)$ is ....
[-7 - 2i; 7 – 2i; -7 + 2i; 7 + 2i]

13. The product of eigen values of a matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is ....

[-1; 0; 1; 2]

14. If U and V are vector spaces with dim U = 3 and dim V = 2, then the dimension of the set L(U, V) of all linear maps from U to V, is ....

[1; 3; 5; 6]

15. If $A = \begin{bmatrix} 1+i & -i & 2 \\ 4+i & 1-i & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1-i \\ 1 & 2 \\ 1-i & i \end{bmatrix}$, then AB is ....

[$\begin{bmatrix} 2-3i & 2 \\ 1-i & 7-5i \end{bmatrix}$; $\begin{bmatrix} 2+3i & 2 \\ 1-i & 7-5i \end{bmatrix}$; $\begin{bmatrix} 2-3i & 2 \\ 1+i & 7-5i \end{bmatrix}$; $\begin{bmatrix} 2-3i & 2 \\ 1-i & 7+5i \end{bmatrix}$]

16. The rank of the matrix $\begin{bmatrix} 3 & 2 & 3 & 1 \\ 4 & 3 & 5 & 2 \\ 2 & 1 & 1 & 0 \end{bmatrix}$ is ....

[1; 2; 3; 4]

17. If the vectors **u** and **v** are orthonormal, then $\|u+v\|^2$ is ....

[0; 1; $\sqrt{2}$; 2]

18. A square matrix A is said to be .... if $A^2 = A$.

[symmetric ; asymmetric ; idempotent ; singular]

19. The kernel of the linear transformation T : $V_3 \to V_2$ defined by $T(x_1, x_2, x_3) = (x_1 - x_2, x_1+x_3)$ is ....

[(1, -1, -1); {(1, -1, -1)}; (1, 1, -1); {(1, 1, -1)}]

20. A linear mapping T : U → V is .... if the nullity of T is the zero space of U, U & V being vector spaces.

[into; one-to one; onto; singular]

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Time : 2 hrs. 30 mins.

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SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define a vector sub-space over a field with example. Show that the span of a non-empty subset S of a vector space V is the subspace of V containing S. Also, in the complex vector space V_2^C, show that (1+i, 1- i) belongs to [(1+i, 1), (1, 1- i)]. [2+3+2]

2. What is meant by the composition of linear maps on vector spaces over the same field of scalars? If T_1, T_2 are linear maps from U to V and S_1 is a linear map from V to W, where U, V, W are vector spaces over the same fields of scalars, then prove that (i) T_1+T_2 is a linear map from U to V, and (ii) $S_1(T_1+T_2) = S_1T_1 + S_1T_2$. [3+2+2]

OR

What is meant by a basis on a vector space? Prove that any set of n linearly independent vectors in n-dimensional vector space V_n is a basis. Also, verify that the set of vectors {(1, 1, 1), (1, -1, 1), (2, 0, 3)} forms a basis for $\Re^3$. [3+2+2]

3. Define a characteristic equation associated to a matrix and its eigen values. Also, find the eigen values and associated vector(s) of the matrix $A = \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$. [2+1+4]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. If T is a one-one linear map from vector space U to another vector space V and $u_1, u_2, \ldots, u_n$ are **LI** vectors of U, then show that $T(u_1), T(u_2), \ldots, T(u_n)$ are also **LI**, symbols have their usual meanings.

5. Prove that a linear transformation is completely determined by its values on the elements of a basis.

OR

Find the matrix representation of the linear transformation T: $\Re^2 \to \Re^2$ defined by $T(x, y) = (x, x - 2y)$ relative to the basis {(1, 0), (-1, 1)}.

6. Show that the mapping T : $V_3 \to V_3$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2, x_2 - x_3, x_1 + x_3)$ is linear. Also, find its nullity $N(T)$.

7. Apply Gram-Schmidt process to orthonormalize the set of linearly independent vectors {(1, 2, 1, (-1, 1, 0), (5, -1, 2)} of V_3.

8. If A and B are matrices of order 3 over the set of complex numbers, then verify that $(AB)^* = B^*A^*$, where A^* denotes the tranjugate of A.

9. Use matrix method to solve the system the system of linear equations: $x - y + 3z = 1$, $2x + y - z = 2$, $3x - y + 2z = 2$. Also, verify the answer.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Find the vector projection of $\vec{B} = \hat{i} + \hat{j} + \hat{k}$ on $\vec{A} = 5\hat{j} - 3\hat{k}$.

11. For non-empty sets A, B, C, prove that $A\times(B\cap C) = (A\times B)\cap(A\times C)$ symbols have usual meanings.

12. Show that the intersection of two subspaces of a vector space is also a vector space.

13. Find the inverse of the real function $g(x) = 3x + 5$.

14. If V has a basis of finite elements then show that every other basis for V has the same elements.