Marks Scored:

KATHMANDU UNIVERSITY
End Semester Examination
August/September, 2017

Level : B.Sc.
Year : I

Course : MATH 103
Semester : II

Exam Roll No. :
Registration No.:

Time: 30 mins.

F. M. : 20
Date SEP 11 2017

SECTION "A"

[10Q × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbols(s).

1. A finite dimensional complex inner product space is ________________ space.

2. For a vector space V with m basis elements, every set of p vectors with p < m is ______

3. Three vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ are ______________ if one of them lies in the plane through the other two.

4. The length of vector $\mathbf{u} = (1, -2, -3, 4)$ is ______________

5. The eigen values of a symmetric matrix are ____________

6. The determinant of $(B - \lambda I)$ in the characteristic equation of the matrix B is _____________ in λ.

7. The direction of the vector projection of **u** along **v** is the same that of **v** if ____________

8. A square matrix A is non-singular if __________________

9. If $T : U \rightarrow V$ is a linear mapping with dim U = dim V = k, then the nullity of T is ______

10. For a subspace U of a vector space V and a vector v of V, then $\{v\} + U$ is ________ variety.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answer from among the given ones.

11. The positive value of x such that the vectors (2, 5) and $(1, x)$ are linearly dependent, is __
[5; 7; 10; 12]

12. The vector $u = (\alpha, -3, 1)$ and $v = (1, \alpha, 2)$ are orthogonal if α is ______
[- 2; – 1; 1; 2]

13. The product of eigen values of a matrix $\begin{bmatrix} 2 & 5 \\ 4 & 1 \end{bmatrix}$ is ________

[-18; -12; - 8; - 4]

14. For subspaces U and W of a vector space V with dimU = 5, dim W = 3 and dim(U $\cap$W) = 2, then dim(U+W) is ________, symbols have usual meanings.

[3; 4; 5; 6]

15. If $C = \begin{bmatrix} 1+i & -i & 2 \\ 4+i & 1-i & 0 \end{bmatrix}$ and $D = \begin{bmatrix} 0 & 1-i \\ 1 & 2 \\ 1-i & i \end{bmatrix}$, then CD is ________

[$\begin{bmatrix} 2+3i & 2 \\ 1-i & 7-5i \end{bmatrix}$; $\begin{bmatrix} 2-3i & 2 \\ 1+i & 7-5i \end{bmatrix}$; $\begin{bmatrix} 2-3i & 2 \\ 1-i & 7+5i \end{bmatrix}$; $\begin{bmatrix} 2-3i & 2 \\ 1-i & 7-5i \end{bmatrix}$]

16. If T : $V_3 \to V_2$ and S : $V_3 \to V_2$ are two linear mappings defined by $S(x_1, x_2, x_3) = (x_1 - x_2, x_2 - x_3)$ and $T(x_1, x_2, x_3) = (2x_1, x_2 - x_3)$ then (S + T)(1, 1) is ________

[(-2, -2); (2, -2); (-2, 2); (2, 2)]

17. The scalar projection of (3, 1) onto (1, 2) is ________

[2; $\sqrt{5}$; 3; 5]

18. A square matrix A is said to be ______________ if $A^2 = I$, an identity matrix.

[symmetric ; asymmetric ; idempotent ; involutory]

19. The kernel of the linear transformation T : $V_3 \to V_2$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2, x_1 - x_3)$ is ______________

[(1, -1, -1); {(1, -1, 1)}; (1, 1, -1); {(1, 1, -1)}]

20. The value of the determinant of a square matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 1 & 3 & 2 \end{bmatrix}$ is ________

[- 10; -5; 5; 10]

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F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define a basis on a vector space with example. Prove that any set of n linearly independent vectors in n-dimensional vector space V_n forms a basis. Also, extend the set {(3, -1, 2)} to a basis for V_3. [2+3+2]

2. What is meant by the composition of linear maps on vector spaces over the same field of scalars? If T is a linear map from U to V and S is a linear map from V to W, where U, V, W are vector spaces over the same field of scalars, and if S and T are non-singular then prove that (i) ST is non-singular (ii) $(ST)^{-1} = T^{-1} S^{-1}$. [2+3+2]

OR

Define a linear transformation over vector space. If T is a one-one linear map from vector space U to another vector space V and if and $u_1, u_2, \ldots, u_n$ are **LI** vectors of U, then prove that $T(u_1), T(u_2), \ldots, T(u_n)$ are **LI**, symbols have their usual meanings. Also, show that the mapping T defined on 2-space V_2 by $T(x_1, x_2) = (x_1 - x_2, x_1 + x_2)$. [2+3+2]

3. Define a characteristic equation associated to square matrix and its eigen value(s). Also, find the eigen values and associated eigen space of the matrix $A = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$. [2+2+3]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. If S is a nonempty subspace of a vector space V, show that the span [S] of S is the smallest subspace of V containing S.

5. If T: U →V is a non-singular linear transformation, U & V being vector spaces. Then, show that its inverse T^{-1} is linear, one-one and onto.

OR

State and prove the Rank-Nullity Theorem of finite dimensional vector space.

6. Show that the set V_3 of all 3-tuples is a vector space with usual addition and scalar multiplication defined on it.

7. Apply Gram-Schmidt Process to orthonormalize the set of linearly independent vectors {(1, 1, 1), (0, 1, 2), (2, 1, 1)} of V_3.

8. If A and B are matrices of order 3 over the set of complex numbers, then verify that (AB)* = B*A*, where A* denotes the tranjugate of A.

9. Use matrix method to solve the system the system of linear equations: $2x - y + 3z = 9$, $x + y + z = 6$, $x - y + z = 2$.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Find the vector projection of $\vec{B} = \hat{i} + 3\hat{j} + 5\hat{k}$ on $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$.

11. Show that $W = \{(x_1, x_2, x_3) \in V_3 : x_1 = 0\}$ is a subspace of V_3.

12. Prove that the real function g defined by $g(x) = 4x + 7$ is bijective.

13. Verify that the set $\{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ forms a basis for V_3.

14. For the matrices $A = \begin{bmatrix} 1 & i \\ i & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & i \\ -i & -1 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.