

KATHMANDU UNIVERSITY
End Semester Examination
September 2024

Level : B.E./BIT
Year : I
Time : 2 hrs. 30 mins.

Course : MATH 101
Semester : I
F. M. : 55

13 SEP 2024

SECTION "C"
[3Q. × 7 = 21 marks]

Attempt *All* questions.

1. State the first derivative test for rise and fall of a function in the interval. If $f(x) = x^{\frac{4}{3}} - 4x^{\frac{1}{3}}$ then investigate rise, fall and extreme value(s) of it and also plot the graph base on above observation. [2+4+1]

OR

- Prove that the differentiability of a function at a point implies continuity at that point. Show that $f(x) = |x|$ is not differentiable at $x = 0$. Find the equation of tangent and normal to the curve $y = x^3$ at $(1, 1)$. [2+2+3]
2. State the first and second fundamental theorem of calculus. Sketch the region enclosed by the curve $y = 2\sqrt{x}$, $1 \leq x \leq 2$, and find the area of the surface generated by revolving the curve it about x-axis. [2+1+4]
3. Define consistent and inconsistent systems of linear equations. Describe the solutions of the following system. [2+5]

$$x_1 + 4x_2 - 2x_3 = 6; \quad 2x_1 + 3x_2 + x_3 = 10; \quad 3x_1 + x_2 + 5x_3 = 14.$$

SECTION "D"
[6Q. × 4 = 24 marks]

4. Evaluate $\frac{dy}{dx}$ (*ANY TWO*):
- i. $y = \frac{x^2-1}{2x-3}$ ii. $y^2 = x^2 + \sin xy$ iii. $ye^x = \ln(x+y)$
5. Evaluate the following (*ANY TWO*):
- i. $\int \frac{1}{x^2+6x+5} dx$ ii. $\int e^x \cos x dx$ iii. $\int_0^a \frac{1}{x\sqrt{x^2-1}} dx$
6. State Rolle's theorem .verify if the function $f(x) = x^2 + 1$ satisfies mean value theorem in the interval $[1,4]$. If so, find the point(s).
7. Find the area of the region bounded by the curve $y^2 = 4ax$ and $x^2 = 4ay$.

OR

Find the area of solid generated by revolution of the arc of the parabola $y^2 = 4ax$ bounded by its latus rectum about x axis.

P.T.O.

8. Test the convergent and divergent of the series $\sum_{i=1}^{\infty} \left(\frac{n^n}{n!}\right)$

9. Find the spanning set $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Evaluate: $\lim_{x \rightarrow 0} \frac{2\cos x - 2 + x^2}{x^4}$

11. Examine the point of discontinuity of the function $f(x) = \frac{x^2 + x^3}{x^2 + 5x + 6}$

12. If $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ is defined by $T(x, y, z) = (x + y + z, 0)$, then show that T is linear.

13. Find the matrix A such that $W = colA$ where $W = \begin{bmatrix} 6a - b \\ a + b \\ -7a \end{bmatrix} : a, b \in \mathbb{R}$.

14. State the Sandwich theorem, by using it find the value of $\lim_{x \rightarrow 0} f(x)$ if $\sqrt{2 - 2x^2} \leq f(x) \leq \sqrt{2\cos x}$

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Marks Scored:

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Exam Roll No. :

Time: 30 mins.

F. M. : 10

Registration No.:

Date : 13 SEP 2024

SECTION "A"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. If f and g both are odd functions defined on \mathbb{R} , then fg function is _____.
2. $\lim_{x \rightarrow 3^+} \frac{|x|}{x} =$ _____, where $[\cdot]$ denotes its usual meaning.
3. Find the Domain and range of the following function $F(x) = \frac{|x-3|}{x-3}$ is _____.
4. The slope of the curve $y = x^2$ at $(1,1)$ is _____.
5. If $y = \int_1^{x^2} \cos t \, dt$, then $\frac{dy}{dx} =$ _____.
6. The volume of the solid obtained by rotating bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$ is _____.
7. If the series is convergent and whose sum of the n - term is $\frac{2n}{3n+5}$ then, the limit is _____.
8. If there are more variables than equations in a homogeneous system of linear equations, then the system has _____ solution(s).
9. The scalar 7 is an eigenvalue of A if and only if the equation $Ax =$ _____.
10. Let V, W be two vector spaces over the same field F . Then, a transformation $T: V \rightarrow W$ is linear if and only if $T(au_1 + bu_2) =$ _____ for u_1, u_2 in V .

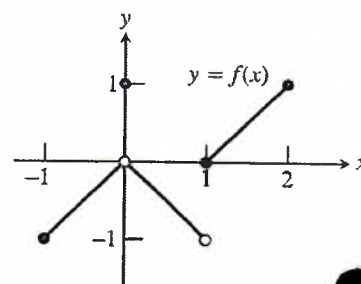
SECTION "B"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. If a graph of a function $y = f(x)$ rises from left to right, then _____.
 [$f'(x) = 0$; $f'(x) < 0$; $f'(x) > 0$; $f''(x) > 0$]

12. Which of the following statements about the function $y = f(x)$ graphed alongside is true? _____.

[$\lim_{x \rightarrow 0} f(x) = 0$; $\lim_{x \rightarrow 0} f(x) = 1$;
 $\lim_{x \rightarrow 1} f(x) = 0$; $\lim_{x \rightarrow 1} f(x) = 1$]



13. Let $f(x)$ and $g(x)$ be positive for x sufficiently large, then f grows _____ g when $x \rightarrow \infty$ if $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 0$.

[slower than; faster than; at the same rate as; none]

14. How many point(s) to the curve $y = x^2 - 4x + 3$ where tangent parallel to x-axis? _____.
 [1; 2; 3; 4]

15. Length of the curve $y = \left(\frac{x}{2}\right)^{\frac{2}{3}}$ from $0 \leq x \leq 2$ is _____.
 [2.27 2.37 2.47 2.55]

16. The number e is that number in the domain of the natural logarithm then $\ln(e) = \int_1^e \frac{1}{t} dt =$

[e $\ln e$ $\log 10$ 1]

17. Let $\sum a_n$ be a series with positive terms, and suppose that $\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \rho$. Then, the series converge if _____.
 [$\rho > 1$; ρ is infinite; $\rho = 1$; $\rho < 1$]

18. If the vectors y be in span $\{v_1, v_2, v_3\}$ where $v_1 = (1, -1, -2)$, $v_2 = (5, -4, -7)$, $v_3 = (-3, 1, 0)$ in R^3 , and $y = (-4, 3, k)$ then $k =$ _____.

[3; 4; -4; 5]

19. The null space of a matrix A is the set Nul A of all the solution of the equation _____.

[$Ax = 0$; $Ax = b$; $A = bx$; $Ax \neq 0$]

20. The scalar λ is characteristic root of the matrix A if _____ is singular.

[A ; λI ; $\lambda - AI$; $A - \lambda I$]