

12. The dimension of null space of the matrix $B = \begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is _____.
 [0; 1; 2; 3]
13. The column of a square matrix U are linearly independent if _____.
 [$\det(U) = 0$; $\det(U) \neq 0$; $\text{rank}(U) = 0$; $\text{rank}(U) \neq 0$]
14. The critical point of the function $f(x)$ if $f'(x) = (x - 4)x^{-\frac{1}{3}}$ is _____.
 [0; 4; 0, 4; 0, -4]
15. The angle between the curves $y = x^2$ and $y = x$ at $x = 0$ is _____.
 [$\frac{\pi}{2}$; $\frac{\pi}{4}$; $\frac{\pi}{8}$; 0]
16. The average value of the function $f(x) = 4 - x^2$ on $[0, 3]$ is _____.
 [-1; 0; 1; 2]
17. The graph of the function $y^2 = x$ is increasing about _____.
 [y-axis; x-axis; line $y = x$; origin]
18. The linear approximation of the function $f(x) = \cos x$ at $x = \frac{\pi}{2}$ is _____.
 [$-x + \frac{\pi}{2}$; $x + \frac{\pi}{2}$; $\frac{\pi}{2}$; 1]
19. The graph of the curve $y = x^2 - x$ is increasing in the interval _____.
 [$(-\infty, 1)$; $(-\infty, \frac{1}{2})$; $(\frac{1}{2}, \infty)$; $(1, \infty)$]
20. The alternating p-series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$ converges for the value(s) of p satisfying _____.
 [$p > 0$; $p < 0$; $0 < p \leq 1$; $p > 1$]

KATHMANDU UNIVERSITY
End Semester Examination [C]
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Level: B.E./B.Sc./B.Tech.
Year : I
Time : 2 hrs. 30 mins.

Course : MATH 101
Semester: I
F. M. : 55

SECTION "C"

[3 Q. × 7 = 21 marks]

1. State Mean Value Theorem for the function defined on $[a, b]$ and explain it geometrically. How this theorem is related to the Rolle's Theorem? Evaluate the value of a, m and b when the function $f(x) = \begin{cases} 3, & x = 0 \\ -x^2 + 3x + a, & 0 < x < 1 \\ mx + b, & 1 \leq x \leq 2 \end{cases}$ satisfies the hypothesis of the mean value theorem on the interval $[0, 2]$? [3+4]

OR

- Define absolute extreme values of the function $y = f(x)$. Find the interval on which $f(x) = -x^3 + 12x + 5, -3 \leq x \leq 3$. is increasing and decreasing. Where does the function assume extreme values and what are these values? [2+2+3]
2. State first and second fundamental theorems of integral calculus? Illustrate each theorem with an example. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and bounded below by the x -axis and the line $y = x - 2$. [2+2+3]
3. Define basis vectors. Write a standard basis vector in \mathbb{R}^2 . Show that the given vectors $\vec{u}_1 = (2, 3, -4), \vec{u}_2 = (4, 8, -5), \vec{u}_3 = (-2, -5, 0)$ are a basis for \mathbb{R}^3 . Write a vector $\vec{b} = (8, 6, 3)$ as a linear combination of $\vec{u}_1, \vec{u}_2, \vec{u}_3$. [1+1+4+1]

SECTION "D"

[6Q. × 4 = 24 marks]

4. Find $\frac{dy}{dx}$ (ANY TWO):
- a) $\sinh x - \cos y = x^2 y$
- b) $y = \sin^{-1} \sqrt{\frac{1 - \cos x}{2}}$
- c) $x = e^{\log \cos 4\theta}, y = e^{\log \sin 4\theta}$
5. Evaluate the following (ANY TWO):
- a) $\int_0^{\pi/2} \frac{dx}{2 + \cos x}$
- b) $\int_0^3 \frac{dx}{(x-1)^{2/3}}$
- c) $\int \frac{x^2}{\sqrt{9-x^2}} dx$

6. Find the length of the function $y = \left(\frac{x}{2}\right)^{2/3}$ from $x = 0$ to $x = 2$.

OR

Find the volume of the solid generated by the region bounded by the curve $y = x^2$ and the line $y = 1$ about the line $y = 2$.

7. State the Ratio Test. Investigate the convergence of the series $\sum_{n=1}^{\infty} \frac{n 2^n (n+1)!}{3^n n!}$
8. Find the possible asymptotes of the function $f(x) = \frac{2x^2-3}{7x+4}$ and sketch the graph.
9. Solve the system of linear equations
 $3x_1 + 8x_2 - 13x_3 = -9$, $5x_1 + 12x_2 - 19x_3 = -11$, $x_1 + 2x_2 - 3x_3 = -1$ by using row rank method.

SECTION "E"

[5Q. \times 2 = 10 marks]

10. Evaluate $\lim_{x \rightarrow 1^+} \frac{1}{x^{1-x}}$.
11. Determine the row rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 5 & 8 \end{bmatrix}$.
12. Solve the initial value problem: $\frac{ds}{dt} = 1 + \cos t$, $s(0) = 4$.
13. Show that the transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ define by $T(x, y) = (x, x + y, 2y)$ is linear.
14. Find the area of the region bounded by the graph of the curve $y = x^2$, and the lines $y = 0$ and $x = 1$.