KATHMANDU UNIVERSITY
End Semester Examination
March, 2025

Marks Scored:

Level : B.E./B.Sc.
Year : I

Course : MATH 101
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : 17 MAR 2025

SECTION "A"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. If the graph of $y = |x|$ is shifted 2 units to the right and 1 unit down, then equation of the curve is ________________.

2. If $\lim_{x \to a} f(x) = 5$ and $\lim_{x \to a} g(x) = -2$ then $\lim_{x \to a} \frac{f(x)}{f(x)-g(x)} =$ ________________.

3. The range of the function $f(x) = \frac{|x|}{x}$ is ________________.

4. The linearization of $f(x) = \cos x$ at $x = \frac{\pi}{2}$ is ________________

5. If $y = \int_1^{\sqrt{x}} \sin u \, du$, then $\frac{dy}{dx} =$ ________________.

6. The volume of the solid generated by the revolution of the curve $y = f(x)$ about the x-axis and the ordinates $x = a$ and $x = b$ is ________________.

7. The limit of the sequence whose nth term is $a_n = \sqrt[n]{n^2}$ ______________.

8. If there are more variables than equations in a homogeneous system of linear equations, then the system has ____________ solution(s).

9. The characteristic equation of $A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$ is ____________.

10. The transformation $T: R^n \to R^m$ is said to be linear if $T(\alpha\vec{u} + \beta\vec{v}) =$ ____________ for all $\vec{u}$ and $\vec{v}$ in R^n.

SECTION "B"

[10 Q.× 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The function $f(x) = \frac{\cos x}{x^2}$ is symmetric about ________________.
[x-axis; y-axis; origin; about the line $y = x$]

12. Length of the curve $y = f(x)$ from $x = a$ to $x = b$ is ________________.
[$\int_a^b \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$; $\int_a^b \sqrt{1 + \frac{dy}{dx}}\, dx$;
$\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$; $\int_a^b \sqrt{1 + \frac{dx}{dy}}\, dy$]

13. Suppose u and v are functions of x that are differentiable at $x = 0$ and that $u(0) = 5, u'(0) = -3, v(0) = -1, v'(0) = 2$, then at $x = 0$, $\frac{d}{dx}\left(\frac{u}{v}\right) =$ ________________.
[-5; 2; 3; -7]

14. The horizontal asymptote of the graph of the function $f(x) = \frac{1}{x^2-1}$ is ________________.
[$y = 0$; $x = 1$; $x = -2$; $x = \pm 1$]

15. The function $f(x) = -x^2$+6x-10 has. Local maximum at________________.
[$x = 1$; $x = -1$; $x = 0$; $x = 3$]

16. If the average value of a continuous function f on the interval [-2, 2] is $\frac{1}{4}$, then
$\int_{-2}^{2} f(x)dx =$ ________________.
[0, 1, 2, 4]

17. Let $\sum a_n$ be a series with positive terms, and suppose that $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \rho$. Then, the series converges if ________________.
[$\rho > 1$; ρ is infinite; $\rho = 1$; $\rho < 1$]

18. The sum of the series $\sum_{n=0}^{\infty} \frac{2^n+5}{3^n} =$ ________________.
[7/2; 21/3; 5/3; 21/2]

19. If one row in an echelon form of an augmented matrix is [0 0 0 0| 1], then the associated linear system is ________________.
[Consistent; Inconsistent;
Unique solution; Infinitely many solutions]

20. The dimension of null space of the matrix $A = \begin{bmatrix} 1 & 2 & 6 & -5 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ ________________.
[1 ; 2; 3; 4]

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SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define differential coefficient of $f(x)$ at $x = a$. Prove that the differentiability of a function at a point implies continuity at that point. By differentiating $x^2 - y^2 = 1$ implicitly, show that $\frac{dy}{dx} = \frac{x}{y}$. Then show that $\frac{d^2y}{dx^2} = -\frac{1}{y^3}$. [1+3+3]

2. Define horizontal and vertical asymptotes of the graph of the function. Find all asymptotes of the curve $y = \frac{2x+3}{5x+7}$ and then sketch the graph. [2+2+3]

3. Define echelon and reduced echelon form a matrix. Find the general solution of the linear system whose augmented matrix is

$$\begin{bmatrix} 0 & 3 & -6 & 6 & 4: & -5 \\ 3 & -7 & 8 & -5 & 8: & 9 \\ 3 & -9 & 12 & -9 & 6: & 15 \end{bmatrix}.$$

Also, find the parametrically represented solution with any two specific solutions (if exists). [2+4+1]

OR

Define vector subspace of a vector space V over the field F. Show that the subset

$S = \{(x, y, z) \in R^3 : x - 2y + 5z = 0\}$ of R^3 is a subspace of R^3. Find the basis vectors and dimension of S. [1+4+2]

SECTION "D"
[6Q. × 4 = 24 marks]

4. Show that $f(x) = \frac{x^2+x-2}{x^2-1}$ has a continuous extension to $x = 1$ and find that extension.

5. Evaluate the following (***ANY TWO***):
i. $\int \frac{1}{\sqrt{e^{2x}-6}} dx$ ii. $\int e^x \cos x \, dx$ iii. $\int_0^\infty \frac{1}{1+x^2} dx$

P.T.O.

6. State Rolle's theorem. Verify mean value theorem for the function $f(x) = x^2 + 2x - 1$ in the interval $[0,1]$.

OR

Define $\epsilon - \delta$ definition of limit of function at $x = a$. Find $\delta > 0$ if $f(x) = \frac{3}{2}x - 1$, $a = 1$, $\epsilon = 0.01$.

7. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

OR

The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

8. Test the convergent and divergent of the series $\sum_{n=1}^{\infty} \frac{(n+3)!}{3!\ n!\ 3^n}$.

9. Let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$, find the eigenvalues and eigenvectors of A.

SECTION "E"
[5 Q. × 2 = 10 marks]

10. What are indeterminate forms? Evaluate: $\lim_{x \to \infty} \left(x - \sqrt{x^2 + x}\right)$.

11. Find an equation for the line tangent to the curve $x = 2\cos t, y = 2\sin t$ at $t = \frac{\pi}{4}$.

12. If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is defined by $T(x, y) = (x - y, 2y)$, then show that T is linear transformation.

13. Describe all solutions of the homogenous system: $10x_1 - 3x_2 - 2x_3 = 0$.

14. For what value of h is $\overrightarrow{v_3}$ in Span$\{\overrightarrow{v_1}, \overrightarrow{v_2}\}$ if $\overrightarrow{v_1} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$, $\overrightarrow{v_2} = \begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix}$ and $\overrightarrow{v_3} = \begin{bmatrix} 5 \\ -7 \\ h \end{bmatrix}$?