KATHMANDU UNIVERSITY
End Semester Examination
March/April, 2025

Marks Scored:

Level : B.E./BIT
Year : I

Course : MATH 101
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : 02 APR 2025

SECTION "A"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The domain of f is A and domain of g is B then the domain of fg is _____.

2. $\lim_{x\to\frac{5}{2}}\left[\frac{x}{2}\right] =$ _____. where [ ] denote it's usual meaning

3. The function $y = |x|$ is differentiable at every point of x except _____.

4. The column of a matrix A are linearly independent if and only if the echelon form of A has _____.

5. The vertical asymptotes of the function $y = \frac{2x}{x-3}$ *is* _____.

6. The critical point of $y = x^2 - 2x$ is _____.

7. If the series is convergent and whose sum of the n-th term is $y = \frac{2n}{3n+5}$, then the limit is _____.

8. If a matrix A has n columns, then rank A + dim(Null A) = _____.

9. The scalar 7 is an eigenvalue of A, if and only if the equation $Ax =$ _____.

10. If a set contains more vectors than there are entries in each vector, then the set is _____.

SECTION "B"

[10 Q.× 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. If $f(x) = -\sqrt{x}$ shifted right by 3 units, then the new function is given by _____.
[$-\sqrt{x-3}$, $\sqrt{x-3}$, $\sqrt{x}+3$, $-\sqrt{x}+3$]

12. If f' changes from positive to negative at c, then f has:_____ at c
[Local maxima, Local minima, Absolute minima, No extreme values]

13. The interval in which function $f(x) = x^2 - 4x + 5$ is increasing is _____
[$(0,\infty)$, $(2,\infty)$, $(-\infty,2)$, $(4,\infty)$]

14. At x=1, the graph of the function $y = 4x^2 - 2x$ is _____
[*Concave up,* Concave down, Stationary, Decreasing]

15. Which of the following is the same as the definite integral $\int_a^b f(x)\,dx$: _____
[$\int_a^b f(x)da$, $\int_a^b f(x)db$, $-\int_b^a f(x)dx$, $-\int_a^b f(x)dx$,]

16. The improper integral $\int_0^\infty \frac{dx}{x^2+4}$: _____
[Diverges to infinity, Converges to zero, Converges to $\frac{\pi}{4}$, Converges to $\frac{\pi}{2}$]

17. The series $\sum n^{-p}$ is convergent iff _____.
[$p < 1$, $p > 1$, $p = 1$, $p \leq 1$]

18. If one row in an echelon form of an augmented matrix is [0 0 0 5] then the associated linear system has _____.
[exactly one solution, two solution,
infinitely many solution, no solution]

19. The maximum rank of a 3 × 3 matrix is: _____
[0, 1, 2, 3]

20. The scalar λ is a characteristic root of the matrix A if _____ is singular
[A, λI, $\lambda - AI$, $A - \lambda I$]

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SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define continuity of a function $f(x)$ at a point x=a. Discuss different types of discontinuity with examples. Prove that differentiable functions are continuous at the point but converse need not be true. [1+3+3=7]

2. State the first and second fundamental theorem of calculus. Find the area of the surface generated by revolving the curve $y= 2\sqrt{x}$, $1 \leq x \leq 2$ about the x -axis. [2+5=7]

OR

Define Roll's theorem and Mean value theorem. Show that the equation $x^3+3x+1 = 0$ has exactly one real solution. Find the value(s) of c that satisfies the equation
$f'(c) = \frac{f(b)-f(a)}{b-a}$
for the function $f(x) = x^3 - x^2$ in the interval [-1, 2]. [2+2+3=7]

3. Define column space of a matrix. Find the null space and column space of a matrix A, given by:

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

Also, find their dimensions. [2+3+2=7]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Find $\frac{dy}{dx}$ (***ANY TWO***):

a. $y = \sqrt{\cos(2x+1)}$ b. $y^2 - x^2 = \sin(xy)$ c. $x^m y^n = (x+y)^{m+n}$

5. Evaluate (***ANY TWO***):

(a) $\int e^x \cos(x)\, dx$

(b) $\int \frac{e^{\tan^{-1}(x)}}{1+x^2}\, dx$

(c) $\int_2^3 \frac{x^2}{x-1}\, dx$

P.T.O.

6. State ratio test for the convergence of an infinite series and Test the convergence of the series: $\frac{2}{3!} - \frac{2^2}{5!} + \frac{2^3}{7!} - \frac{2^4}{9!} + \ldots$

7. Find the volume of the solid of revolution of curve $y = \sqrt{x}$, and the line $y = 1$, $x = 4$, about the line $y = 1$.

OR

Find the length of the curve $y = x^{3/2}$, from $x = 0$ to $x = 4$.

8. For what value of h will y be in span of $\{v_1, v_2, v_3\}$. If:

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, \quad y = \begin{bmatrix} -4 \\ 3 \\ h \end{bmatrix}$$

9. Find eigenvalues and eigenvectors of matrix:

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

SECTION "E"
[5 Q. × 2 = 10 marks]

10. Find all asymptotes of $y = \frac{x^2-1}{x^2+1}$.

11. Evaluate $\lim_{x \to 1} \frac{x^2-1}{x-1}$.

12. Find the domain of $y = y = -\sqrt{x}$

13. Show that the transformation $T : R^2 \to R^2$ defined by $T(x,y) = (x - y, x + y)$ is a linear transformation.

14. Let

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Then determine whether $\{v_1, v_2, v_3\}$ is linearly dependent or independent.