

KATHMANDU UNIVERSITY
End Semester Examination [C]
July, 2024

Marks Scored:

Level : B.Arch.

Course : MATH 105

Year : I

Semester : I

Exam Roll No. :

Time: 30 mins.

F.M : 20

Registration No.:

Date : 18 JUL 2024

SECTION "A"

[10 Q. \times 1 = 10 marks]

Fill in the blanks space(s) by most appropriate word(s) or symbol(s):

1. Every equation of first degree in x, y, z represents _____
2. The range of the function $g(x, y) = \text{Cos}(xy)$ is _____
3. The value of $\frac{d}{dx}(\log(\text{Tan } x))$ at $\pi/4$ is _____
4. A function $y = f(x)$ is _____ function if $f(-x) = f(x)$ for all x .
5. The function $f(x) = \text{Cos}(\frac{1}{x})$ has _____ type discontinuity at point $x = 0$.
6. If $F(x) = \int_0^{x^2} \text{Cos } t \, dt$, the value of $F'(x)$ at $x = 0$ is _____
7. $\lim_{x \rightarrow 3^-} \frac{[x]}{[x]} =$ _____
8. The value of $8 \Gamma(\frac{7}{2})$ is _____
9. The first order differential equation of $x^2 + y^2 = 16$ is _____
10. The perpendicular distance from $(2, 3, 4)$ to the plane $3x - 2y + 6z + 4 = 0$ is _____ units.

SECTION "B"

[10 Q. \times 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answer from among the given ones.

11. The z-intercept of the plane $2x - 3y + 4z = 12$ is _____
[-2 ; 2 ; 3 ; 4]
12. The linear approximation of $f(x) = x^2 + 4$ near $x = 1$ is $f(x) \approx$ _____
[$2x + 1$; $2x + 2$; $2x + 3$; $2x + 4$]

13. The average value of $f(x) = \sqrt{x}$ on the interval $[0, 4]$ is _____
 [1/3; 2/3; 4/3; 5/3]
14. The normal to the curve $C: x^2 + 3xy + y^2 = 5$ at point $(1, 1)$ is _____
 [-1; 0; 1; 2]
15. The radius of the sphere $x^2 + y^2 + z^2 + 4x - 2y + 2z = 10$ is _____ unit.
 [2; 3; 4; 5]
16. The graph of the function $y = x^2 - 1$ is symmetric about _____
 [y-axis; x-axis; line $y = x$; origin]
17. The oblique asymptote to the curve $y = \frac{x^2-3}{2x-4}$ is _____
 [$y = \frac{x}{2} + 1$; $y = \frac{x}{2} - 1$; $x = 3$; $y = 3$]
18. $\lim_{x \rightarrow \infty} (\sqrt{x} - \sqrt{x-3}) =$ _____
 [-3; 0; 3; ∞]
19. The integrating factor of the differential equation $\frac{dy}{dx} + y \cot x = \operatorname{Cosec} x$ is

 [$\sin x$; $\cos x$; $\sin x + C$; $\cos x + C$]
20. The value of $\int_{-1}^1 |x| dx$ is _____.
 [-1; 0; 1; 2]

KATHMANDU UNIVERSITY
End Semester Examination [C]
July, 2024

Level : B.E./B.Sc.

Year : I

Time : 2 hrs. 30 mins.

Course : MATH 101

Semester : I

F.M. : 55

SECTION "C"

[3 Q. × 7 = 21 marks]

1. State the Mean Value Theorem and explain its relationship to Rolle's Theorem. Describe the geometric interpretation of the Mean Value Theorem. Show that $f(x) = 2x^2 - 3x + 1$ meets the hypotheses of the Mean Value Theorem on the interval $[0,2]$, and then verify the theorem. [2+2+3=7]

OR

Discuss the curve $y = x^4 - 4x^3$ with respect to rise and fall of the curve, concavity, points of inflection, and local maxima and minima. Use this information to sketch the curve. [4+3=7]

2. Define null space and column space. Find null space and column space of the matrix A given by $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$. Also find their dimensions. [2+3+2=7]

3. Define continuity of the function at a point. Check the continuity of the function defined by $f(x) = \begin{cases} 3-x, & x < 2 \\ 2, & x = 2 \\ \frac{x}{2}, & x > 2 \end{cases}$ at $x = 2$. If the function is not continuous at $x = 2$, redefine the function to make it continuous there. Discuss different types of discontinuities. [1+2+1+3=7]

SECTION "D"

[6 Q. × 4 = 24 marks]

4. Evaluate $\frac{dy}{dx}$ (any two):

i. $y = \sqrt{3x^2 - 4x + 5}$

ii. $x = 2 \cos t, y = 3 \sin t$

iii. $y = e^x \sin x$

5. Evaluate the following (any two):

i. $\int_2^3 \frac{x^2}{x-1} dx$

ii. $\int \sec^3 x dx$

iii. $\int \frac{\sin(\tan^{-1} x)}{1+x^2} dx$

6. Find the equations for the lines that are tangent and normal to the curve $x^2 + 2y^2 = 9$ at $(1, 2)$.

P.T.O.

7. Find the area of the region enclosed by the line $y = 2x$ and the curve $y = x^2$.

OR

Find the volume of a solid generated by revolving the curve $y = \sqrt{x}$, with $1 \leq x \leq 2$ about x -axis.

8. State the ratio test for the convergence of the infinite series $\sum a_n$. Check the convergence of $\sum_{n=1}^{\infty} \frac{2^{n+5}}{3^n}$.
9. What is echelon form? Solve the following system of equations using row operations:
 $2x_1 - 3x_2 + 2x_3 = 1$, $x_2 - 4x_3 = 8$, $5x_1 - 8x_2 + 7x_3 = 1$.

SECTION "E"

[5 Q. \times 2 = 10 marks]

10. Evaluate $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$ using L' Hospital's Rule.
11. Find the horizontal and vertical asymptotes of the graph of $f(x) = \frac{x}{x-4}$.
12. Determine whether the mapping $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T(x, y) = (x + y, 4x + 5y)$ is a linear transformation.
13. Find the eigenvalues of the matrix, $A = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$.
14. When a circular plate of a metal is heated in an oven, its radius increases at the rate of 0.1 cm/min. At what rate is the plate's area increasing when the radius is 50 cm?

12. The interval in which function $f(x) = x^2 - 4x + 5$ is increasing is _____.
 [(0, ∞); (2, ∞); (-∞, 2); [4, ∞)]
13. At $x = 1$, the graph of the function $f(x) = 4x^2 - 2x$ is _____.
 [concave up; concave down; stationary; decreasing]
14. $\lim_{x \rightarrow \infty} x \sin \frac{1}{x} =$ _____.
 [0; 1; ∞; not defined]
15. If $f(x) = \begin{cases} 0, & 0 \leq x < 1 \\ 1, & 1 \leq x \leq 2 \\ 0, & 2 < x \leq 3 \end{cases}$, then $\int_0^3 f(x) dx =$ _____.
 [0; 1; 2; 3]
16. Length of the curve $x = \cos t, y = \sin t, 0 \leq t \leq 2\pi$ is _____.
 [π ; 2π ; 3π ; 6π]
17. For a(an) _____ function $f(x)$, $\int_{-p}^p f(x) dx = 0$.
 [even; odd; increasing; constant]
18. Let $\sum a_n$ be a series with positive terms and suppose that $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \rho$, then the series $\sum a_n$ converges if _____.
 [$\rho > 1$; $\rho = 1$; $\rho < 1$; $\rho = \infty$]
19. A system of linear equations is _____ if it has either one solution or infinitely many solutions. _____.
 [consistent; inconsistent; parallel; collinear]
20. What is the maximum rank of a 3×3 matrix? _____.
 [1; 2; 3; 4]