KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

Marks Scored:

Level : B.E./B.Sc./B.Tech.
Year : I
Course : MATH 101
Semester : I
Exam Roll No. :
Time: 30 mins.
F. M. : 20
Registration No.:
Date : 11 JUL 2023

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbol(s).

1. The range of the function $y = \sqrt{1 - x^2}$ is _____.

2. The number of vectors in a basis for the vector space over a field is _____.

3. The function $y = |x|$ is differentiable at every point of x except_____.

4. The average value of the function $f(x) = \sqrt{x}$ with respect to x from $x = 0$ to $x = 4$, is _____.

5. If $y = \int_0^{x^2} \sin t \, dt$, then $\frac{dy}{dx} =$ _____.

6. In the row echelon form of a square matrix, all the elements below the leading _____. are zeros.

7. The sum of the eigen values of the matrix $A = \begin{bmatrix} 3 & 6 & 7 \\ 0 & 4 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ is _____.

8. $\frac{d}{dx}(sec^{-1}x) =$ _____.

9. The scalar projection of (-1, 1, 1) on (3, -2, 1) is _____.

10. If the function $y = a\,x^2 - 4x + 1$ has local minimum value at $x = 1$, then the value of a is _____.

SECTION "B"
[10Q × 1=10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. If vectors (1, -2, 3) and (2, k, 2) in $\mathbb{R}^3$ are orthogonal, then k = _____
[- 2, 0; 2; 4]

12. The linear approximation of $f(x) = x^2 + 4$ near $x = 1$ is $f(x) \approx$ _____
[- $2x$; $2x - 3$; $2x$; $2x + 3$]

13. The value of $\int_{-1}^{1} |x| \, dx$ is _____
[0; 1; 2; 3]

14. $\lim_{x \to 0} \frac{(1-\cos x)}{x+x^2} =$.....
[0; $\frac{1}{2}$; 1; 2]

15. The improper integral $\int_0^{\infty} \frac{dx}{x^2+4}$ _____
[diverges to infinity; converges to 0; converges to $\frac{\pi}{2}$; converges to $\frac{\pi}{4}$]

16. The solution set of the inequality $|2x - 3| \leq 1$ is _____
[$[0, 1]$; $[1, 2]$; $[1, 3]$; $[0, 3]$]

17. The slope of the tangent to the curve C : $x^2 + xy - y^2 = 1$ at (2, 3) is _____
[$\frac{3}{4}$; $\frac{5}{4}$; $\frac{7}{4}$; $\frac{9}{4}$]

18. A linear system is called _____ if all of the constant terms are zero.
[consistent; inconsistent; homogeneous; non-homogeneous]

19. For what value of k, do the following equations $2x - 3y = 1$, $4x - 6y = k$ have infinitely many solutions?
[$k = 0$; $k = 1$; $k = 2$; $k \neq 2$]

20. A vector transformation $T: V \to W$ is linear if and only if for every $x, y \in V$ and any scalars α, β _____.
[$T(x + y) = T(x) + T(y)$; $T(\alpha x) = \alpha T(x)$;
$T(\alpha x + \beta y) = \alpha T(x) + \beta T(y)$; $T(x - y) = T(x) - T(y)$]

KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

11 JUL 2023

Level : B.E./B.Sc./B.Tech.
Year : I
Time : 2 hrs. 30 mins.

Course : MATH 101
Semester : I
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Write the condition for the existence of the limit of a function at a point. Prove that a derivable function at a point is also continuous at that point. Also, find the continuous extension of $f(x) = \frac{x^2-16}{x^2-3x-4}$ at $x = 4$. [1+2+3]

2. State Mean Value Theorem for the function defined on $[a, b]$ and explain it geometrically. Also, verify it for $f(x) = x^2 + 2x - 1$ on [0, 1]. [2+2+3]

OR

Write the second derivative test for the extreme values of the function. Also, find the extreme values of $f(x) = x^4 - 8x^2 + 16$ defined on [-3, 3]. [2+5]

3. Define the characteristic equation of the matrix and its associated Eigen values. Also, find the Eigen space of the matrix $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$. [2+2+3]

SECTION "D"
[6Q. × 4 = 24 marks]

4. Find $\frac{dy}{dx}$ *(ANY TWO)*.
   a. $y = x \sin^{-1}\sqrt{1-x^2}$ b. $y^2 - x^2 = \sin(xy)$
   c. $x = 2a\tan\theta$; $y = a\sec^2\theta$

5. Evaluate the following integrals *(ANY TWO)*.
   a. $\int_0^1 \frac{1}{(x+1)(x^2+1)}dx$ b. $\int e^x \cos x\, dx$ c. $\int_0^\infty x\, e^{-x^2}\, dx$

6. Find the volume of solid generated by revolving the region bounded by $y = x - x^2$ and $y = 0$ about x- axis.

7. Find the asymptotes of the function $y = \frac{x^2-4}{x-1}$ and also sketch the curve.

8. Use matrix method to solve the following system of linear equations
   $4x - y - 2z = 15$; $-x + 2y + 3z = 5$; $5x - 7y + 9z = 8$
   Also verify the solution.

OR

State the Ratio Test for the convergence of an infinite series. Test the convergence of the series $\sum \frac{3^n+2}{5^n}$.

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9. Show that the given vectors $u_1 = (2,3,4)$, $u_2 = (4, 8,5)$ and $u_3 = (2, 5, 0)$ are basis for $\mathbb{R}^3$.

SECTION "E"

[5Q. × 2 = 10 marks]

10. Find the normal to the curve $x^2 + xy - y^2 = 1$ at $(2, 3)$.

11. Evaluate the limit: $\lim_{x\to 0} \frac{\sin x - x}{x^3}$.

12. Solve the initial value problem $\frac{dy}{dx} = \tan x$, $y(0) = 2$.

13. A spherical balloon is inflated with helium at the rate of $100\pi\ ft^3\ /min$. How fast is the balloon's surface area increasing at the instant the radius is $5ft$?

14. Show that the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x - y, x + y)$ is a linear transformation.