Marks Scored:

KATHMANDU UNIVERSITY
End Semester Examination
February/March, 2019

Level : B.E./B. Sc./B. Tech.
Year : I

Course : MATH 101
Semester : I

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Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date 06 MAR 2019

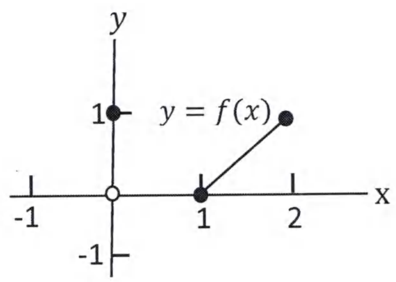
SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blanks space(s) by writing the most appropriate word(s) or symbol(s).

1. The range of function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \sqrt{4-x}$ is ________________.

2. The function whose value at any number x is the greatest integer smaller than or equal to x is called ________________________.

3. Find $\lim_{x \to 1^+} f(x)$ from the following graph of the function $f(x)$ ____________________ .



4. The graph of the function $f(x) = x^3 - 3x$ has horizontal tangents at the points where $x =$ __________.

5. If the function f is integrable on $[a, b]$, its average value over $[a, b] =$ ________________ .

6. If $y = \int_0^{x^2} \sin t \, dt$, then $\frac{dy}{dx} =$ ___________________.

7. If a mapping $T: V \to W$ is linear, then $T(0) =$ _____________.

8. The sum of the infinite series $\sum_{n=1}^{\infty} 0.5^n$ =___________.

9. If there are more variables than equations in a homogeneous system of linear equations, then the system has _____________________solution(s).

10. Eigenvalues of the matrix $\begin{bmatrix} 1 & 5 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$ are ______________.

SECTION "B"

[10 Q.× 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. If f' changes from positive to negative at c, then f has____________________ at c.
[Local maxima; Local minima; No extreme values; Absolute minima]

12. $\lim\limits_{x \to 0} \frac{1-\cos x}{x+x^2} =$ ____________________.
[0; $\frac{1}{2}$; $-\frac{1}{2}$; 2]

13. A function f is continuous at left end point $x = a$ of its domain if
[$\lim\limits_{x \to a} f(x) = f(a)$; $\lim\limits_{x \to a^+} f(x) = f(a)$;
$\lim\limits_{x \to a^-} f(x) = f(a)$; $\lim\limits_{x \to a^-} f(x) = \lim\limits_{x \to a^+} f(x) = f(a)$]

14. The number of vertical asymptotes on the graph of the function $f(x) = \frac{x^3-1}{x^2-1}$ is ____________________ .
[zero; one; two; three]

15. Length of the curve $x = 3\cos t, y = 3\sin t, 0 \le t \le 2\pi$ is ________________.
[π; 2π; 3π; 6π]

16. $\int_{-1}^{1} |x| dx =$____________________.
[-2; −1; 1; 2]

17. $s = 0.8t^2$ is the equation for free fall of a body on the moon. Find the body's average velocity over the interval $0 \le t \le 5$ is ________________.
[0.8; 2.5; 4; 5]

18. If the vectors $(-2, 2, 4), (-1, 1, k), (1, 0, 3)$ in R^3 are linearly dependent, then $k =$______.
[-1; 1; 2; 3]

19. Rank of the matrix $\begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 2 & 0 & 3 \\ 0 & 0 & -1 & 1 \end{bmatrix}$ is____________________.
[1; 2; 3; 4]

20. The sequence $\{a_n\}_{n=1}^{\infty}$ with $a_n = \left(\frac{n+1}{n-1}\right)^n$ converges to ________________.
[0; e; e^{-2}; e^2]

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Time : 2 hrs. 30 mins.

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Semester : I
F.M. : 55

SECTION "C"

[3 Q.× 7 = 21 marks]

1. State the second derivative test for the concavity of a function over an interval. If $f(x) = x^4 - 4x^3 + 10$; find the intervals on which f is increasing and the intervals on which f is decreasing and find where the graph of f is concave up and where it is concave down. Sketch the graph for the function. [1+3+3]

OR

State Mean Value Theorem for the function defined on $[a, b]$ and explain it geometrically. How is this theorem related to Rolle's Theorem? If the function $f(x)$ defined by

$$f(x) = \begin{cases} -3 & \text{for} \quad x = -1 \\ x^2 - 2cx + a & \text{for} \quad -1 < x < 0 \\ mx + c & \text{for} \quad 0 \le x \le 1 \end{cases}$$

satisfies the hypotheses of the Mean Value Theorem on the interval $[-1, 1]$, evaluate a, m and c. [3+1+3]

2. Derive formula to evaluate the definite integral as a limit of a sum. Evaluate the integral $\int_0^1 (2x^2 + 3x + 1)dx$ by using that formula. [4+3]

3. Define vector subspace and linear combination of the vectors. Describe the span of the vectors $\vec{u}_1 = (1, 0, 1)$, $\vec{u}_2 = (1, 1, 3)$, $\vec{u}_3 = (2, 3, 8)$. Write each vector in the span as the linear combination of the given vectors. [2+4+1]

SECTION "D"

[6 Q.× 4 = 24 marks]

4. Evaluate $\frac{dy}{dx}$ (*ANY TWO*):

   i. $ax^2 + 2hxy + by^2 = 1$ ii. $y = (4x + 3)^4 (x + 1)^{-3}$ iii. $y = \left(1 + \tan^4 \frac{x}{12}\right)^3$

5. Evaluate the following (*ANY TWO*):

   i. $\int \frac{(1+\sqrt{x})^{\frac{1}{3}}}{\sqrt{x}} dx$ ii. $\int_0^{\frac{\pi}{6}} (1 - \cos 3x) \sin 3x \, dx$ iii. $\int e^x \sin x dx$

6. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

OR

Find the area of the region enclosed by the curve $y^2 - 4x = 4$ and the line $4x - y = 16$.

7. Define the continuous extension of a function f(x) at $x = c$. Show that the function $f(x) = \frac{x^2+x-6}{x^2-4}$ has a continuous extension at $x = 2$.

8. State Ratio Test for the convergence of an infinite series. Test the convergence of the series

$$\frac{2}{3!} - \frac{2^2}{5!} + \frac{2^3}{7!} - \frac{2^4}{9!} + \cdots$$

9. Solve the following system of linear equations

$$\begin{aligned} x_1 + 2x_2 + 2x_3 + 3x_4 &= 0 \\ 2x_1 + 4x_2 + 3x_3 + 7x_4 &= 0 \\ x_1 + 2x_2 + x_3 + 4x_4 &= 0 \end{aligned}$$

SECTION "E"

[5 Q.× 2 = 10 marks]

10. Evaluate: $\lim_{x\to\infty} x^{\frac{1}{1-x}}$.

11. Find all asymptotes of the graph of the function $f(x) = \frac{x^2-3}{2x-4}$.

12. Solve the initial value problem $\frac{dy}{dx} = \cos x,\ y(0) = 4$.

13. Show that $\vec{x} = (2, 3)$ is the eigenvector of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

14. A spherical balloon is inflated with helium at the rate of $100\pi\, ft^3/min$. How fast is the balloon's surface area increasing at the instant when the radius is $5ft$?