KATHMANDU UNIVERSITY
End Semester Examination
February/March, 2018

Marks Scored:

Level : B.E/B.Sc./B.Tech.
Year : I

Course : MATH 101
Semester: I

Exam Roll No.:

Time: 30 mins.

F.M. : 20

Registration No.:

Date MAR 0 8 2018

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by most appropriate word(s) or symbol(s).

1. The mean value theorem says that at some interior points of an interval, the instantaneous rate of change must equal to a(n) _____.

2. A line $x = a$ is a vertical asymptote of the graph if _____ _____.

3. If f is continuous on [a,b] and $u(x)$ and $v(x)$ are differentiable functions of x whose value lie in [a,b] then $\frac{d}{dx}\int_{u(x)}^{v(x)} f(t)dt =$ _____.

4. If the density of a thin rod is $\delta(x)$ then the moment about origin on $[a, b]$ is _____.

5. $\int a^x dx =$ _____.

6. The area under the curve $y = \frac{1}{1-x}$ from $x = 0$ to $x = 1$ is _____ _____.

7. A series that converges but does not converge absolutely _____.

8. A linear system is _____ if it has no solution.

9. A consistent system with fewer equations than variables has _____ _____.

10. The dimension of null space is called _____.

SECTION "B"
[10 Q.×1=10 marks]

Fill in the blank space(s), DO NOT TICK, by choosing the most appropriate answer from among the given ones.

11. The function has no derivative at a point where the graph has _____.
[Vertical tangent; Horizontal tangent; Oblique tangent; Continuity]

12. If $s = -t^3 + 3t^2 - 3t$ be the position of a moving body on a coordinate line, then the body's instantaneous rate of change at $t = 2$ is _____ .
[3; -3; 2; -2]

13. The function $y = x^2$ on [0,2] has absolute maximum value _____ .
[0; 1; 2; 4]

14. The average value of $f(x) = 3x^2 - 3$ on [0,1] is _____ .
[-2; 2; -3; 0]

15. $\lim_{x\to 0} \frac{1-\cos x}{x+x^2} =$ _____ .
[0; 1; 1/2; 2]

16. $\lim_{n\to\infty} \frac{x^n}{n!} =$ _____ .
[∞; 1; 0; e^x]

17. The set of all vectors $\vec{X}$ such that $A\vec{X} = \vec{0}$ is called _____ .
[Null space; Column space; span; linear combination]

18. If every column in reduced (row) echelon form has leading 1 then the vectors are _____ .
[Linearly dependent; Linearly independent; Span; Basis]

19. The eigenvalues of the matrix $A = \begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix}$ are _____ .
[2, 4; 2, 3; 3, 4; 0, 4]

20. A linear system is called _____ if all of the constant terms are zero.
[consistent; inconsistent;
homogeneous; non homogeneous]

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Year : I
Time : 2 hrs. 30 mins.

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Semester: I
F.M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define different types of asymptotes of a function. Find all asymptotes of $y = \frac{x^2-4}{x-1}$ and sketch the curve. [1+1+1+2+2]

OR

Define concavity and state second derivative test to find concavity. Find the intervals on which the function $y = x^4 - 2x^2$ is increasing/decreasing, concave up/down and sketch the curve. [1+1+1.5+1.5+2]

2. State second fundamental theorem of integral calculus and use it to find the area between the region $y = x^3 - 3x^2 + 2x, 0 \le x \le 2$ and the x-axis. Also find the area of the region in the first quadrant that is bounded by the curves $y = x^2, x + y = 2$ and x-axis. [1+2+4]

3. Define row rank and nullity of a matrix. Find a basis for the null space, rank and nullity of the matrix [1+1+3+1+1]

$$A = \begin{bmatrix} 5 & 10 & -5 \\ 3 & 7 & -2 \\ 2 & 7 & 1 \end{bmatrix}.$$

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Find dy/dx (any *two*):

a) $y = \sqrt{\frac{x}{x+1}}$ b) $5x^{4/5} + 10y^{6/5} = 15$ c) $r = \sin(\theta + \sqrt{\theta + 1})$

5. Evaluate the following integrals (any *two*):

a) $\int \frac{1}{\sqrt{t}} \cos(\sqrt{t} + 3) dt$ b) $\int_1^2 \frac{2^{\ln x}}{x} dx$ c) $\int \tan^{-1} y \, dy$

6. Find the local extreme values of the function $f(x) = x^4 - 8x^2 + 16$.

7. Find the area of the surface generated by revolving the curve $x = 2\sqrt{4 - y}, 0 \le y \le \frac{15}{4}$ about y-axis.

OR

Find the volume of solid generated by revolving the region bounded by the curves $y = 4 - x^2$ and $y = 2 - x$.

8. Decide whether the series $\sum_{n=1}^{\infty} \frac{e^n}{1+e^{2n}}$ converges or diverges.

9. Find the limit: $\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Find the slope and equation of tangent of the curve $x^2 + xy - y^2 = 1$ at (2, 3).

11. For the limit $\lim_{x \to 10} \sqrt{19 - x} = 3$, find a $\delta > 0$ that works for $\epsilon = 1$.

12. Solve the system of equations $\begin{matrix} x_1 + 3x_2 + 4x_3 = 3 \\ 2x_1 + 6x_2 + 9x_3 = 8 \end{matrix}$.

13. Find the eigenvalues of $\begin{bmatrix} -11 & 3 \\ -27 & 7 \end{bmatrix}$.

14. Show that the transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x, y, z) = (x + y + z, 0)$ is linear transformation.