KATHMANDU UNIVERSITY
End Semester Examination [C]
November/December, 2023

30 NOV 2023

Level : B.E./B.Sc./B.Tech. | Course : MATH 101
Year : I | Semester : I
Time : 2 hrs. 30 mins. | F. M. : 55

SECTION "C"

[3 Q. × 7 = 21 marks]

1. Define Sandwich Theorem of limit. Use Sandwich theorem to evaluate $\lim_{x \to 0} u(x)$, if the function $u(x)$ satisfies the condition $1 - \frac{x^2}{4} \le u(x) \le 1 + \frac{x^2}{2}$ for all $x \neq 0$. Write the types of discontinuities of a function $f(x)$ with an example of each. [1+2+4]

2. Define Roll's theorem and Mean value theorem. Show that the equation $x^3 + 3x + 1 = 0$ has exactly one real solution. Find the value(s) of c that satisfies the equation
$$f'(c) = \frac{f(b) - f(a)}{b - a}$$
for the function $f(x) = x^3 - x^2$ in the interval $[-1, 2]$. [2+2+3]

OR

Sketch the graph of the function $\frac{x^2+4}{2x}$ using all necessary descriptions.

3. Define linearly independent vectors. Check whether or not the set of vectors $\vec{v_1} = \begin{bmatrix}1\\2\\1\\4\end{bmatrix}, \vec{v_2} = \begin{bmatrix}0\\1\\1\\1\end{bmatrix}, \vec{v_3} = \begin{bmatrix}2\\0\\1\\7\end{bmatrix}$ are linearly independent? Also, find the basis of the vectors $\begin{bmatrix}1\\2\\1\end{bmatrix}, \begin{bmatrix}-1\\-1\\1\end{bmatrix}, \begin{bmatrix}1\\4\\5\end{bmatrix}, \begin{bmatrix}3\\4\\-1\end{bmatrix}, \begin{bmatrix}0\\1\\2\end{bmatrix}, \begin{bmatrix}4\\9\\6\end{bmatrix}$. [1+3+3]

SECTION "D"

[6Q. × 4 = 24 marks]

4. Find $\frac{dy}{dx}$ (any two):

   a. $x^2 - y^2 = 2x$ b. $y = \sin x . \cos x$ c. $y = e^{x^2 - x}$

5. Evaluate the following integrals (any two):

   a. $\int \frac{\sin x}{\cos^2 x} dx$ b. $\int \frac{1}{(x+2)(x-5)} dx$ c. $\int_0^3 |x - 2| \, dx$

6. When a circular plate of metal is heated in an oven, its radius increases at the rate of $0.01 cm/min$. At what rate is the plate's area increasing when radius is $50cm$?

7. State fundamental theorem of integral calculus (Part I) and evaluate $\int_x^5 3t \sin t \, dt$.

8. The region bounded by the curve $y = \sqrt{x}$, the x-axis, and the line $x = 4$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

OR

The line segment $x = 1 - y,\ 0 \le y \le 1$, is revolved about the y-axis to generate the cone. Find its lateral surface area of that cone.

9. Show that the p–series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots$, where p is a real constant converges for $p > 1$ and diverges for $p \le 1$.

<u>SECTION "E"</u>

[5Q. × 2 = 10 marks]

10. For a given $\epsilon = 0.01 > 0$, if the limit of the function $f(x) = x + 1$ at $c = 4$ is $L = 5$, find the value of $\delta > 0$.

11. Does the graph $f(x) = \begin{cases} x \sin\frac{1}{x} & x \ne 0 \\ 0, & x = 0 \end{cases}$ have a tangent line at the origin? Give reasons for your answer.

12. Find the length of the graph of $f(x) = \frac{1}{3}(x^2 + 2)^{\frac{3}{2}},\quad 0 \le x \le 3$.

13. Define row echelon and reduced echelon form of a matrix with suitable examples of each.

14. Find $X = (x_1, x_2)$ in which the matrix $A = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix}$ transforms X into $Y = (5, -3)$.

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SECTION "A"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbol(s).

1. The value of $\lfloor -0.7 \rfloor$ is ________________

2. The domain of the function $f(x) = \sqrt{1-x}$ is ________________________

3. The derivative of $|x|$ for $x < 0$ is ____________________

4. The average value of $-\frac{x^2}{2}$ on $[0,3]$ is ___________________

5. The area between the graph of $f(x) = x^2 - 4$ and the x – axis over $[-2, 2]$ is _____________

6. If f' is continuous on $[a, b]$, then the arc length of the curve $y = f(x)$ from $(a, f(a))$ to $(b, f(b))$ is __________________

7. The sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ is _____________________

8. If the constant terms of a system of linear equations are all zero, then the system of linear equations is called ________________

9. The indexed set of vectors $\{v_1, v_2, v_3, \dots, v_p\}$ in $\mathbb{R}^n$ is said to be ___________________ if the vector equation $x_1v_1 + x_2v_2 + \cdots + x_pv_p = 0$ has only the trivial solution.

10. The Eigenvalues of the matrix $A = \begin{bmatrix} 2 & -1 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & -2 \end{bmatrix}$ are ____________

SECTION "B"

[10Q × 1=10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The value of $\lim_{x \to 64} \frac{x^{\frac{2}{3}} - 16}{\sqrt{x} - 8}$ is ________________________.

$[\frac{4}{3};$ $\quad \frac{8}{3};$ $\quad \frac{3}{4};$ $\quad \frac{3}{8}]$

12. The point of inflection of the function $f(x) = 2x^3 - 5\,x$ is at ______________.
[$x = 5$; $x = 2$; $x = 0$; $x = -2$]

13. If f' changes sign from negative to positive at c, then f has ______________ at c.
[maxima; minima;
no extreme values; both maxima and minima]

14. If $f(x)$ is an even function, then ______________.
[$f(-x) = f(x)$; $f(-x) = -f(x)$; $f(x) = x$; $f(x) = -x$]

15. Which of the followings is same as the definite integral $\int_a^b f(x)\,dx$? ______________.
[$\int_a^b f(x)\,da$; $\int_a^b f(x)\,db$; $\int_b^a f(x)\,dx$; $-\int_b^a f(x)\,dx$]

16. The volume of a solid of revolution of the curve $y = f(x)$ and the x – axis from $x = a$ to $x = b$ is the integral ______________.
[$\int_a^b [f(x)]^2 dx$; $\int_a^b \pi[f(x)]^2 dx$; $\int_b^a [f(x)]^2 dx$; $\int_b^a \pi[f(x)]^2 dx$]

17. Let $\sum a_n$ be any series and suppose that $\lim_{n \to \infty} \left|\frac{a_{n+1}}{a_n}\right| = \rho$, then the series diverges if ______________.
[$\rho > 0$; $\rho < 0$; $\rho > 1$; $\rho < 1$]

18. System of equations having infinitely many solutions is called ______________.
[consistent and dependent; consistent and independent;
inconsistent and dependent; inconsistent and independent]

19. The set of vectors $\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}2\\0\\1\end{bmatrix}, \begin{bmatrix}4\\2\\5\end{bmatrix}, \begin{bmatrix}-1\\4\\6\end{bmatrix}$ are ______________.
[linearly independent; linearly dependent;
span; linear transformation]

20. Eigenvector corresponding to the greater Eigenvalue of the matrix $A = \begin{pmatrix}2 & 1\\1 & 2\end{pmatrix}$ is ______________.
[$\begin{bmatrix}1\\-1\end{bmatrix}$; $\begin{bmatrix}-1\\-1\end{bmatrix}$; $\begin{bmatrix}-1\\1\end{bmatrix}$; $\begin{bmatrix}1\\1\end{bmatrix}$]