KATHMANDU UNIVERSITY
End Semester Examination
March 2025

Level : BDSc Year : I	05 MAR 2025	Course : DSMA 115 Semester : I
Exam Roll No. :	Time: 30 mins.	F. M. : 20
Registration No.:		Date : 05 MAR

Marks Scored:

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. Starting with the graph of $y = \sin x$, the equation of the graph that results from shifting it 2 units down becomes ____________________.

2. $\lim_{x \to 0} \frac{e^x - 1}{x} =$ ____________________.

3. The graph of the function $f(x) = \frac{5x^2+8x-3}{3x^2+2}$ has the line __________ as a horizontal asymptote.

4. The tangents to the graphs of the function $y = 2x^2 - 4x + 1$ are parallel to the x-axis at the point(s), $x =$ ____________________.

5. If $y = \int_0^x (5t^2 - 3)dt$, then $\frac{dy}{dx} =$ ____________________.

6. The length of the curve, $y = 2x^{3/2}$ over the interval $[0, 1]$ is __________________ units.

7. The nth term of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \cdots$ is $a_n =$ ____________________.

8. The infinite series $\sum_{n=0}^{\infty} \frac{1}{n^p}$ converges if ____________________.

9. The Maclaurin series for $y = e^x$ is ____________________.

10. The value of $\Gamma\left(\frac{3}{2}\right) \times \Gamma\left(\frac{1}{2}\right)$ is ____________________, where the symbols have their usual meanings.

SECTION "B"

[10 Q.× 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The domain of the function defined by $y = \sqrt{1 - x^2}$ is ______________.

[$(0, \infty)$; $[1, \infty)$; $[-1, 1]$; $(-\infty, \infty)$]

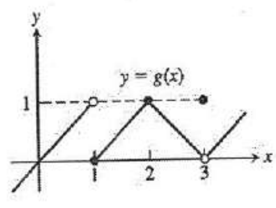
12. The value of the greatest integer function, $\lfloor 1.9 \rfloor =$ ______________.

[1; 1.5; 2; none]

13. $y = x^2$ grows ______________ than $y = 2x^3 + 2$.

[slower than; faster than; at the same rate as; none]

14. Which of the following statements about the function $y = g(x)$ graphed alongside is true? ______________.



[$\lim_{x \to 0} g(x) = 1$; $\lim_{x \to 1} g(x) = 0$; $\lim_{x \to 2} g(x) = 1$; $\lim_{x \to 3} g(x) = 1$]

15. The linearization of $f(x) = 2x^3 - 5x + 2$ at $a = 2$ is $L(x) =$ ______________.

[$19x - 38$; $19x - 30$; $12x - 5$; $6x - 5$]

16. $\int_{-1}^{1} |x| dx =$ ______________.

[0; $\frac{1}{2}$; 1; 2]

17. The average value of a function $y = 3x^2$ over the interval $[1, 2]$ is ______________.

[8; 7; $\frac{7}{3}$; 21]

18. Which of the following relation is not true? ______________.

[$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx$; $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{2\Gamma(m+n)}$; $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$; $\beta(m,n) = 2\int_0^{\frac{\pi}{2}} \cos^{2m-1}\theta \sin^{2n-1}\theta \, d\theta$]

19. Suppose that $a_n > 0$ and $b_n > 0$ for all $n \geq N$ (N and integer). If $\lim_{n \to \infty} \frac{a_n}{b_n} =$ ______________ and $\sum b_n$ converges, then $\sum a_n$ converges.

[0; 1; $c > 0$; ∞]

20. For what value of x, the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(3x)^n}{n}$ converge? ______________.

[$|x| < 1$; $|x| > 1$; $|x| < 3$; $|x| < \frac{1}{3}$]

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05 MAR 2025

SECTION "C"

[2 Q. × 8 = 16 marks]

1. Define critical point, the point of inflection and concavity with examples. Discuss the concavity, point of inflection and local maxima and minima of the function $y = x^4 - 4x^3 + 10$. Use this information to sketch a graph of this function. [3+3+2]

OR

State Mean Value Theorem. How is this theorem related to Rolle's Theorem? Describe the geometrical interpretation of the Mean Value Theorem. For what values of a, m, and b does the function

$$f(x) = \begin{cases} 3, & x = 0 \\ -x^2 + 3x + a, & 0 < x < 1 \\ mx + b, & 1 \leq x \leq 2 \end{cases}$$

satisfy the Mean Value Theorem hypotheses on the interval [0, 2]? [1+1+2+4]

2. Define improper integral, discuss its types, and evaluate $\int_1^\infty \frac{dx}{1+x^2}$. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$. [4+4]

SECTION "D"

[6Q. × 4 = 24 marks]

3. Discuss the types of discontinuities with examples. Evaluate $\lim\limits_{x\to\infty} x^{1/x}$.

4. Find vertical and oblique asymptotes of the graph of $f(x) = \frac{x^2-3}{2x-4}$. Also, check if any horizontal asymptotes exist.

5. Find the equation of the tangent to the curve $y = 1 + \cos x$ at $x = \frac{\pi}{2}$. The radius of a circular oil slick on the surface of a pond is increasing at the rate of 10 meters/min. At what rates are the circle's circumference and area changing?

6. Evaluate the following (***ANY TWO***):
   a. $\int \frac{x^2}{\sqrt{1-x^3}} dx$
   b. $\int \tan^4 x \, dx$
   c. $\int_0^4 xe^{-x} dx$

P.T.O.

7. Investigate the convergence of the series $\sum_{n=0}^{\infty} \frac{2^n+5}{3^n}$ and $\sum_{n=1}^{\infty} \left(\frac{1}{n+1}\right)^n$.

OR

State comparison test and use the test to investigate the convergence of the series $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^{\frac{3}{2}}}$.

8. Show that $\frac{\beta(m+1,n)}{m} = \frac{\beta(m,n+1)}{n} = \frac{\beta(m,n)}{m+n}$ using the relation between Beta and Gamma functions. Evaluate $\int_0^1 x^{\frac{3}{2}}(1-x)^{5/2}dx$.