KATHMANDU UNIVERSITY
End Semester Examination
December 2024/ January 2025

Level : B.E.
Year : IV
Time : 2 hrs. 30mins.

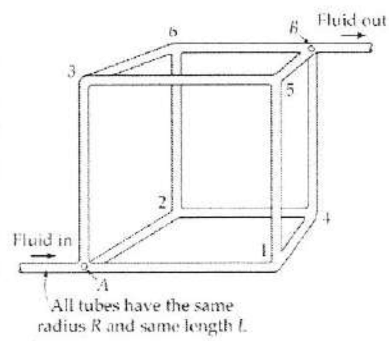
05 - jan. 025

Course : CHEG 401
Semester : I
F. M. : 40

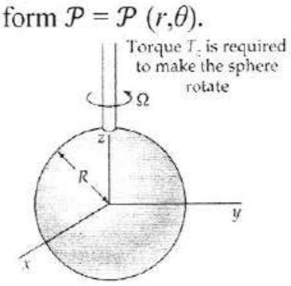
SECTION "B"
[40 marks]

Attempt *ALL* questions. The data or information not given in the questions should be assumed properly.

1. A fluid is flowing in laminar flow from A to B through a network of tubes, as depicted in figure below. By using shell balance at any one of the first set of three pipes splitting from A, obtain an expression for the mass flow rate of the fluid entering that pipe in terms of the total inlet flow rate of w at A. In the equation, use modified pressure drop only at the junction where three pipes split into six pipes. Neglect the disturbances at the various tube junctions. [8]



2. A solid sphere of radius R is rotating slowly at a constant angular velocity Ω in a large body of quiescent fluid (see figure below). Simplify the equations of motion in spherical coordinates (given below the figure) to find the the components of the creeping flow equation of motion. Assume $\mathbf{v} = \boldsymbol{\delta}_\varphi\, v_\varphi(r,\theta)$ and that the modified pressure will be of the form $\mathcal{P} = \mathcal{P}\,(r,\theta)$. [5]



$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r}$$
$$+\mu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2 v_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_r}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial \phi^2}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2\cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$+\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial \phi^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right] + \rho g_\theta$$

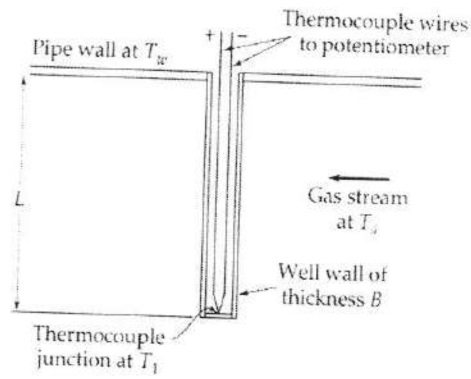
$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi\cot\theta}{r}\right) = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}$$
$$+\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\phi\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial \phi^2} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \phi} + \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right] + \rho g_\phi$$

P.T.O.

3. In figure below, thermocouple is shown in a cylindrical well inserted into a gas stream. Using shell energy balance of a fin structure, prove that $\Theta(\zeta)=\cosh N(1-\zeta)/\cosh N$, where Θ $((T-T_a)/(T_w-T_a))$ is dimensionless temperature, ζ is dimensionless distance (z/L), and $N^2 = hL^2/(kB)$. All other variables have their usual meanings. [8]

For equation $\frac{d^2y}{dx^2} - a^2y = 0$, solution is $y = C_1 \cosh ax + C_2 \sinh ax$

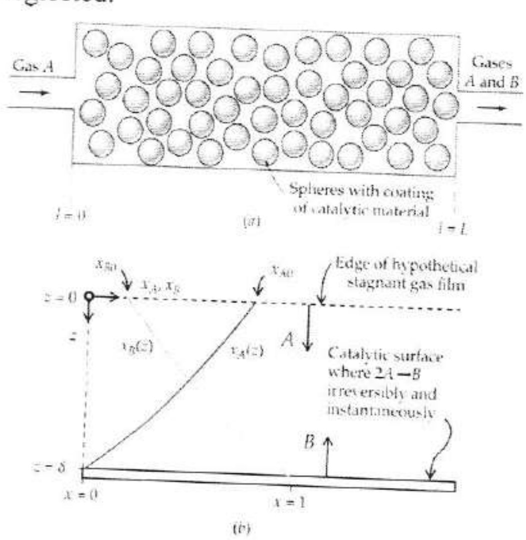
$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$



4. In the energy equation shown below, expand **e** vector and explain what equation is obtained for flow systems without external forces. [3]

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \rho\hat{U}\right) = -(\nabla \cdot \mathbf{e}) + \rho(\mathbf{v} \cdot \mathbf{g})$$

5. A simple model for a heterogeneous catalytic reactor in which a reaction $2A \rightarrow B$ is given in figure below. Each catalyst particle is surrounded by a stagnant gas film through which A has to diffuse to reach the catalyst surface. The product B then diffuses back out through the gas film to the main turbulent stream composed of A and B. Find the local rate of reaction per unit area of catalyst surface by shell mass balance. Use the assumption that A is present in such a small concentration that the convection term can be neglected. [8]



6. In the following equation of continuity for species A in a binary mixture of A and B, describe the conditions under which the equation becomes the Fick's second law of diffusion. [3]

$$c\left(\frac{\partial x_A}{\partial t}+\left(\mathbf{v}^{*}\cdot\nabla x_A\right)\right)=-\left(\nabla\cdot\mathbf{J}_A^{*}\right)+R_A-x_A\left(R_A+R_B\right)$$

OR

Prove that $[\nabla \cdot s\boldsymbol{\delta}] = \nabla s$ where $\boldsymbol{\delta}$ is unit tensor. [3]

7. Water at 10°C enters a heat-exchanger tube having an inside diameter of 25 mm and a length of 3 m. The water flows at 76 L/min. Take viscosity as 0.434 mPa·s and friction factor as 0.0042. Entrance effects are to be neglected, and the properties of water may be evaluated at the arithmetic-mean bulk temperature. We have: $\ln((T_L - T_s)/(T_0 - T_s)) + (h/(\rho v C_p))\times(4L/D) = 0$, where v is velocity, T_L is exit temperature, T_0 is inlet temperature and T_s is wall temperature. C_p = 4.2 kJ/kg·K and k = 0.676 W/m·K. For a constant wall temperature of 99°C, estimate the exit temperature of the water using the Colburn analogy ($\mathrm{St} = (C_f/2)\,\mathrm{Pr}^{-2/3}$). [5]