KATHMANDU UNIVERSITY
End Semester Examination
February, 2025

Marks Scored:

Level : B.Sc.
Year : II

Course : BIMA 206
Semester : II

Exam Roll No. :

Time: 30 mins.

F. M. : 10

Registration No.:

Date : 24 FEB 2025

SECTION "A"

[10Q. × 0.5 = 5 marks]

Fill in the blank space(s) by the most appropriate word(s) or symbol(s).

1. The order of the differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} + x^2y^3 = \sin x$ is ________________.

2. If the general solution of the ODE (IVP) $y' + 4y = 1.4,\ y(0) = 2$ is $y = Ce^{-4x} + 0.35,$ then value of the constant $C =$ __________________.

3. A first order ordinary differential equation is called separable if it can be written into the form ____________________________.

4. If the roots of the characteristic equation are $\lambda = 1$ and $\lambda = 2,$ then corresponding characteristic equation is __________________.

5. If the characteristic equation of the second order homogeneous equation has only one root $\lambda = 2$, then the general solution is written as ______________.

6. The Wronskian of the functions $y_1 = \cos x$ and $y_2 = \sin x$ is ________________.

7. If the approximated solution of an ordinary differential equation $\frac{dy}{dx} = x + y, y(0) = 1$ is defined on the interval $[0, 1]$ by dividing it into 10 equal sub-intervals, then value of step size is __________________.

8. The two sources of error while solving ordinary differential equations using numerical methods are _______________ error and ________________ error.

9. The Runge-Kutta method is _______________ order approximation method to solve the ODE (IVP) $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$ numerically.

10. For the system of first-order linear differential equations $\frac{dx}{dt} = x - y$ and $\frac{dy}{dt} = -x + 2y,$ the corresponding system matrix is __________________.

SECTION "B"

[10 Q. × 0.5 = 5 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones. (***DO NOT TICK THE ANSWER***)

11. The degree of the differential equation $(y')^3 = ((y')^2 + 1)^{1/2}$ is _______________.
    [2   3   6;   1]

12. The integrating factor of the differential equation $\frac{dy}{dx} - y = \sin x$ is ______________.
    [e^{-x}; e^{x};; $-x$; x]

13. The general solution of the differential equation $\frac{dy}{dx} = 2x$ is _____________.
    [$y = x^2 + C$ $y = x + C$ $y = \ln x + C$; $y = e^x + C$]

14. A trivial solution of homogeneous ordinary differential equation is _____________.
    [$y = x$; $y = e^x$; $y = 0$; $y = \ln x$]

15. The logistic growth model for population $y(t)$ of a species at time t is given by ______________.
    [$\frac{dy}{dt} = ry$ $\frac{dy}{dt} = \frac{r}{y}$ $\frac{dy}{dt} = r(1 - \frac{K}{y})y$; $\frac{dy}{dt} = r(1 - \frac{y}{k})y$]

16. The equation __________________ is the characteristic equation of the differential equation $y'' + y' - 6y = 0$.
    [$\lambda^2 + \lambda + 6 = 0$; $\lambda^2 + \lambda - 6 = 0$; $\lambda^2 - \lambda - 6 = 0$; $\lambda^2 - \lambda + 6 = 0$]

17. The local truncation error in the Euler's method is of the order of ________________.
    [$O(h)$; $O(h^2)$; $O(h^3)$; $O(h^4)$]

18. In the autonomous differential equations, the rate of change of the dependent variable depends on the ____________________.
    [independent variables only;
    dependent variables only;
    both independent and independent valuables;
    neither independent nor independent valuables]

19. For a non-homogeneous second order linear ordinary differential equations, a common method to find a particular solution is ______________________.
    [separation of variables;   integrating factor;
    Euler's method ;   variation of parameters]

20. For the system of first-order linear differential equations given in matrix form $\frac{dX}{dt} = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} X$, the eigenvalues of the system matrix are ________________ and ____________.
    [2 and 1;   2 and -1;   -2 and -1;   -2 and 1]

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Time : 2 hrs. 30mins.		F. M. : 40

SECTION "C"

[2 Q. × 8 = 16 marks]

1.

a. Define a first order linear ordinary differential equation. Write few difference between linear and non-linear first order ordinary differential equations. [2]

b. Classify the homogeneous and non-homogeneous first order linear differential equations and explain the solution techniques. [0.5+2]

c. Solve the initial value problem $\frac{dy}{dx} + y = xe^{-x} + 1, y(0) = 1$. [3+0.5]

2.

a. Why do we need numerical methods to solve differential equations? Write a few numerical methods for approximating the solutions of first order ordinary differential equations. [2]

b. Derive the formula for the improved Euler's method to approximate the solution of first order ordinary differential equation $\frac{dy}{dx} = f(x, y)$ with initial condition $y(x_0) = y_0$. [3]

c. Use the improved Euler's method to solve $\frac{dy}{dx} = 2 + x - y$ with initial condition $y(0) = 1$ to find the approximate solutions on the interval $0 \leq x \leq 1$ taking the step size 0.5. [3]

OR

a. Write a system first order ordinary differential equations (ODEs) and discuss when these equations are linear, homogeneous and non-homogeneous. Write a form of linear system of ODEs. [3]

b. Solve the system $\frac{dx}{dt} = -2x + y; \frac{dy}{dt} = -5x + 4y$ with initial conditions $x(0) = 2, y(0) = 3$ by finding the Eigenpairs of the system matrix. [5]

SECTION "D"

[6 Q. × 4 = 24 marks]

3. Plot the direction field of the differential equations $\frac{dy}{dx} = x + y$ on the rectangular region $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$ by dividing the both the intervals on 4 equal parts.

P.T.O.

4. Define the Bernoulli's type differential equation. Solve the differential equation $\frac{dy}{dx} + \frac{1}{3}y = \frac{1}{3}(1-2x)y^4$ with initial condition $y(0) = 1$.

5. The population dynamics of a certain species by Gompertz growth model is given by the differential equation $\frac{dy}{dt} = ry\ \ln\left(\frac{K}{y}\right)$ subject to the initial condition $y(0) = y_0$. Then, solve this equation to find $y(t)$. Find y when $t = 3\ years$ for the given data $r = 0.7$ per year, $K = 80 \times 10^6\ kg$ and $y_0 = 0.25K$.

OR

The field mouse population satisfies the differential equation $\frac{dp}{dt} = \frac{p}{2}$ - 450. Then find the solution is given by $p(t)$ when the initial population of mouse is 850 and find the time (years) at which the population becomes extinct.

6. Solve the second order differential equation $y'' + 4y = x^2 + 3e^x$, $y(0) = 0, y'(0) = 1$ by using the method of undetermined coefficients.

7. Find the approximated solutions of system of ODEs $\frac{dx}{dt} = x - y + xy, \frac{dy}{dt} = 3x - 2y - xy$ with initial conditions $x(0) = 0, y(0) = 1$ at $t = 0.2$ and $t = 0.4$ using the Euler's method.

8. Write the general solution of the given Eigenpairs $\lambda = -1$; $\vec{u} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ of the system matrix A in a given system of ODEs $\frac{dX}{dt} = A$, where $X = \begin{pmatrix} x \\ y \end{pmatrix}$.