KATHMANDU UNIVERSITY
End Semester Examination
March, 2025

Level : B.Sc.
Year : II
Time : 2 hrs. 30 mins.

20 MAR 2025

Course : BIMA 201
Semester : I
F. M. : 40

SECTION "C"
[2 Q. × 8 = 16 marks]

1. Define characteristics equation. Find the eigenvalues of $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. For each eigenvalue, find a basis for eigenspace and diagonalize it. [1+2+3+2]

2. Briefly describe the process of LU factorization of a matrix $A$. Find LU factorization of $A = \begin{bmatrix} 3 & -7 & -2 & 2 \\ -3 & 5 & 1 & 0 \\ -9 & 5 & -5 & 12 \end{bmatrix}$. Use the LU factorization and solve $A\vec{x} = \vec{b}$, where $\vec{b} = \begin{bmatrix} -9 \\ 5 \\ 7 \\ 11 \end{bmatrix}$ [2+3+3]

**OR**

Define determinant of an $n \times n$ matrix $A$. Compute det A, where $A = \begin{bmatrix} 3 & -1 & 2 & -5 \\ 0 & 5 & -3 & -6 \\ -6 & 7 & -7 & 4 \\ -5 & -8 & 0 & 9 \end{bmatrix}$.

Find the adjugate of a matrix $A = \begin{bmatrix} 0 & -2 & -1 \\ 3 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix}$ and find $A^{-1}$. [1+3+4]

SECTION "D"
[8 Q. × 3 = 24 marks]

3. Solve the following system using elementary row operations if consistent:

$$x_1 - 2x_2 - x_3 + 3x_4 = 0$$
$$-2x_1 + 4x_2 + 5x_3 - 5x_4 = 0$$
$$3x_1 - 6x_2 - 6x_3 + 8x_4 = 2$$

4. Find a general solution of the system

$$x_1 + 4x_2 - 5x_3 = 0$$
$$2x_1 - x_2 + 8x_3 = 0$$

5. Let $A = \begin{bmatrix} 1 & -1 & 5 \\ 2 & 0 & 7 \\ -3 & -5 & -3 \end{bmatrix}$ and $w = \begin{bmatrix} -7 \\ 3 \\ 2 \end{bmatrix}$. Determine if $w$ is in Nul A?

**P.T.O.**

6. Find the area of a parallelogram whose vertices are $(-1, 0), (0, 5), (1, -4), (2, 1)$.

7. Find the dimension of null space and column space of $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$.

8. Find the orthogonal projection of $\vec{y}$ onto span$\{\overrightarrow{u_1}, \overrightarrow{u_2}\}$,
where $\vec{y} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}, \overrightarrow{u_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \overrightarrow{u_2} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

9. Find a least square solution of $A\vec{x} = \vec{b}$ where $A = \begin{bmatrix} 1 & -2 \\ -1 & 2 \\ 0 & 3 \\ 2 & 5 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 1 \\ -4 \\ 2 \end{bmatrix}$

**OR**

Find the equation $y = \beta_0 + \beta_1 x$ of the least squares line that best fits the data points: $(2,3), (3,2), (5,1), (6,0)$.

10. Prove that $T: \mathbb{R}^2 \to \mathbb{R}^4$ defined by $T(x_1, x_2) = (2x_2 - 3x_1, x_1 - 4x_2, 0, x_2)$ is a linear transformation.