KATHMANDU UNIVERSITY
End Semester Examination
July/August 2024

Level : B.Sc.
Year : II
Time : 2 hrs. 30mins.

0 2 AUG 2024

Course : BIMA 201
Semester : I
F. M. : 40

SECTION "C"
[2 Q. × 8 = 16 marks]

1. Define row echelon form of a matrix. Solve the following system of equations using elementary row operation to transform the augmented matrix into reduced echelon form and express constant vector b as a linear combination of columns of coefficient matrix A: [1+5+2]

$$3x_2 - 6x_3 + 6x_4 + 4x_5 = -5$$
$$3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 = 9$$
$$3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 = 15$$

2. Define orthonormal set of vectors. State Gram-Schmidt process of orthogonalization.

Find an orthogonal basis for the column space of $A = \begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{bmatrix}$. [1+2+5]

OR

Define eigenvalue of a matrix. Diagonalize the matrix $A = \begin{bmatrix} -3 & 12 \\ -2 & 7 \end{bmatrix}$ and compute A^8. [1+5+2]

SECTION "C"
[8Q. × 3 = 24 marks]

3. Find the LU factorization of $\begin{bmatrix} 2 & -2 & 4 \\ 1 & -3 & 1 \\ 3 & 7 & 5 \end{bmatrix}$.

4. Determine if the set of vectors $\{v_1, v_2, v_3\}$ are linearly dependent or independent. If possible find the linear dependence relation.

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

5. Prove that $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x_1, x_2) = (x_1 - 2x_2, -x_1 + 3x_2, 3x_1 - 2x_2)$ is a linear transformation.

6. Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$.

P.T.O.

7. Solve the following system using Cramer's rule:

$$2x_1 + x_2 + x_3 = 4$$
$$-x_1 + 2x_3 = 2$$
$$3x_1 + x_2 + 3x_3 = -2$$

8. Find a basis for null space of $A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix}$.

9. Let $A = \begin{bmatrix} -6 & 12 \\ -3 & 6 \end{bmatrix}$ and $w = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Determine if w is in Col A. Is w in Nul A?

10. Let $y = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $u = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$. Write y as the sum of two orthogonal vectors, one in Span$\{u\}$ and one orthogonal to u.

OR

Find a least square solution of $Ax = b$ where, $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{bmatrix}, b = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$

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Marks Scored:

Level : B.Sc.
Year : II

Course : BIMA 201
Semester : I
F. M. : 10

Exam Roll No. :

Time: 30 mins.

Date : 0 2 AUG 2024

Registration No.:

SECTION "A"

[10 Q. × 0.5 = 5 marks]

Fill in the blank space(s) by most appropriate word(s) or symbol(s).

1. The variable corresponding to pivot column in the matrix is called ....
2. For a set of vectors $\vec{v_1}, \vec{v_2}, \vec{v_3}, \cdots, \vec{v_p}$ in $\mathbb{R}^n$ and given scalars $c_1, c_2, c_3, \cdots, c_p$, the vector $\vec{y}$ is expressed as a linear combination $\vec{y} =$......................................
3. The homogeneous equation $A\vec{x} = \vec{0}$ has a nontrivial solution if and only if the equation has at least one ....
4. The null space of a matrix A is the set Null A of all solutions to the ....
5. A basis for a subspace H of $\mathbb{R}^n$ is a .... set in H that spans H.
6. A scalar λ is an eigenvalue of an $n \times n$ matrix A if and only if λ satisfies the characteristics equation ....
7. An $n \times n$ matrix A is diagonalizable if and only if A has n linearly ....
8. A set $\{\vec{v_1}, \vec{v_2}, \vec{v_3}, \cdots, \vec{v_p}\}$ is an orthonormal set if it is orthogonal set of ....
9. A set $\{\vec{v_1}, \vec{v_2}, \vec{v_3}, \cdots, \vec{v_p}\}$ is an orthonormal basis for a subspace W of $\mathbb{R}^n$, then $Proj_W \vec{y} =$...............................
10. If A and B are $n \times n$ matrices then det(AB)=...........................

SECTION "B"

[10 Q. × 0.5 = 5 marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answer from among the given ones.

11. A system of equation is inconsistent if it has ....

[exactly one solution ; At least one solution ; Infinitely many solution ; no solution]

12. The columns of a matrix A are linearly independent if and only if the equation $A\vec{x} = 0$ has ....

[trivial solution ; no solution ; Infinitely many solution ; at least one solution]

13. The matrix $\begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$ of a linear transformation represents ....

[horizontal contraction and expansion; horizontal shear ; vertical contraction and expansion ; vertical shear]

14. The rank of a matrix A, denoted by rank A is the dimension of

[null space ; sub space ; column space ; vector space]

15. The determinant of matrix $A = \begin{bmatrix} 3 & -7 & 8 & 9 \\ 0 & 2 & -5 & 7 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ is ....

[12; 6; 4; 2]

16. The area of the parallelogram whose vertices are (0,0), (5,2), (6,4), (11,6) is ....

[8; -8; 16; -16]

17. Let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$ and $\vec{u} = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$. Then $\vec{u}$ is an eigenvector of A corresponding to the eigenvalue ....

[-6; -5; -4; -3]

18. The eigenvalues of the matrix $A = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$ are ....

[$1 + i, 1 - i$; $2 + i, 2 - i$; $2 + 2i, 2 - 2i$; $2 + 3i, 2 - 3i$]

19. The set of least squares solutions of $A\vec{x} = \vec{b}$ coincides with the non empty set of solutions of the normal equations

[$AA^T\vec{x} = A^T\vec{b}$; $A^TA\vec{x} = A^T\vec{b}$; $AA^T\vec{x} = A\vec{b}$; $A^TA\vec{x} = A\vec{b}$]

20. If a unit square is projectd onto X-axis then the standard linear transformation matrix is ....

[$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$; $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$; $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$; $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$]