KATHMANDU UNIVERSITY
End Semester Examination
March/April, 2024 ~~25~~
13 · April - 2025

Marks Scored:

Level : B.Tech.
Year : II

Course : AIMA 201
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 10

Registration No.:

Date :

SECTION "A"

[10Q. × 0.5 = 5 marks]

Fill in the blank space (s) by writing the most appropriate word(s) or symbol(s).

1. The solution of the differential equation $\frac{dy}{dx} = 3y$ with initial condition $y(0) = 2$ is _____.

2. The condition _____ is the necessary and sufficient condition for the ordinary differential equation $M(x,y)dx + N(x,y)dy = 0$ to be exact.

3. The particular solution to the differential equation $y'' + y = 2e^x$, $y_p =$ _____.

4. The equation of the form $x^2y'' + axy' + by = 0, (a, b \text{ are constants})$, is called _____.

5. The system of the equations $y_1' = 2y_1 - y_2$; $y_2' = 3y_1 - 2y_2$ is representing the second order differential equation _____.

6. If y_1 and y_2 are basis to a general solution of second -order ODE, then the Wronskian of y_1 and y_2, $W(y_1, y_2) =$ _____ .

7. The characteristics of the partial differential equation $u_{xx} + u_{xy} - 2u_{yy} = 0$ are _____.

8. The second order PDE $x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} = e^x$ is a _____ type.

9. The Laplace transform of $\cos^2 t$ is _____.

10. The Laplace transform of Dirac's delta (unit impulse) function with impulse $t = a$, i.e. $L\{\delta(t-a)\} =$ _____ for $x > a \geq 0$.

SECTION "B"

[10Q × 0.5 = 5 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The degree of $y' + y = \frac{5}{y'}$ is _____.

[1; 2; 3; 4]

12. The integrating factor of the equation $(1 - x^2)\frac{dy}{dx} - xy = 1$ is _____.

[$\sqrt{1 - x^2}$; $\sqrt{x^2 - 1}$; $\frac{1}{2}\ln(1 - x^2)$; $\sin^{-1} x$]

13. Basis of the second order differential equation $y'' + 2y' + y = 0$ are _____.

[e^x and e^{-x}; e^{-x} and xe^{-x}; e^x and xe^x; e^x and xe^{-x}]

14. The functions e^x and e^{-2x} are the basis of $y'' + ay' + by = 0$ for $a =$ ___ and $b =$ ___.

[2, 1 ; 1, 2 ; −2, 1 ; 1, −2]

15. _____ are the system of linear ODEs corresponding to the second order ODE: $2y'' - 5y' + y = 0$.

[$y_1' = y_2, y_2' = -\frac{1}{2}y_1 + \frac{5}{2}y_2$; $y_1' = y_2, y_2' = -2y_1 + 5y_2$;

$y_1' = y_2, y_2' = \frac{5}{2}y_1 - \frac{1}{2}y_2$; $y_1' = y_2, y_2' = 5y_1 - 2y_2$]

16. A mass of 2 kg is attached to a string. The spring constant is 40. The mass is stated in motion from the equilibrium position with an initial velocity of 1.5 m/sec in the downward direction. Suppose that there is no air resistance and the position is measured in meters and time in second. Then the initial value problem governing the position, $u(t)$ of the mass is_____.

[$u'' + 20u' = 0, u(0) = 0, u'(0) = 1.5$; $u'' + 20u = 0, u(0) = 0, u'(0) = 1.5$;

$u'' + 20u' = 0, u(0) = 1.5, u'(0) = 0$; $u'' - 20u = 0, u(0) = 0, u'(0) = 0$]

17. If $u(x, y) = \lambda\, e^{-y} + \gamma$ is a solution of $u_{xy} = -u_x$, then λ and γ are functions of _____ and _____, respectively.

[x, x; y, y; y, x; x, y]

18. The Laplace transform of t^n is _____.

[$\frac{n!}{s^n}$; $(n + 1)!/s^{n+1}$; $\frac{n!}{s^{n+1}}$; $\frac{n!}{s^{n-1}}$]

19. The inverse Laplace transform $\frac{1}{s(s+2)}$ is _____.

[$\frac{1}{2}(1 - e^{2t})$; $\frac{1}{2}(1 - e^{-2t})$; $\frac{1}{2}(1 + e^{2t})$; $\frac{1}{2}(1 + e^{-2t})$]

20. The convolution $(1 * t) =$ _____.

[$\frac{1}{2}$; $\frac{t}{2}$; $\frac{t^2}{2}$; $\frac{e^t}{2}$]

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Time : 2 hrs. 30 mins. | F. M. : 40

SECTION "C"
[3Q. × 6 = 18 marks]

1.

a. Solve the following equation, Newton's Law of Cooling, by separation of variable.

$$T'(t) = k[T(t) - T_s]$$

where $T(t)$ denote the temperature of the object at time t and T_s the temperature of the surrounding (assumed to be constant throughout). [2]

b. What will be the temperature of the object at $t = 10$ for $T_s = 32°F$, $T(0) = 66°F$ and $k = -0.046187$. [1]

c. Define Bernoulli's equation and hence solve the following logistic equation

$$\frac{dy}{dx} = ay - by^2 \quad (a, b \text{ are positive constant})$$

by reducing into linear form. [1+2]

2. Each of the two tanks contains 200 gal of water, in which initially 100 lb. (Tank T_1) and 200 lb. (Tank T_2) of fertilizer are dissolved. The mixture is kept uniform by stirring. **Fig. 1** describes the balanced equation of inflow, circulation, and outflow of mixing problem. Let y_1 and y_2 are the quantities of fertilizers in Tank T_1 and Tank T_2, respectively.

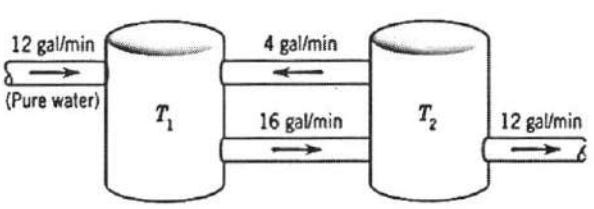

Fig. 1: Mixing Problem

a. Model the above problem in a system of linear differential equations. [2]
b. Solve the system obtained in (a) for y_1 and y_2. [4]

3. If $L\{f(t)\} = F(s)$ exists for $t \geq 0$, then prove that
a. $L\{f'(t)\} = sL\{f(t)\} - f(0)$. [2]
b. $L\{f''(t)\} = s^2L\{f(t)\} - sf(0) - f'(0)$. [1]
c. Using (a) and (b) solve the following boundary value problem.

$$y'' + 9y = \cos 2t\,, y(0) = 1, y\left(\frac{\pi}{2}\right) = -1.$$ [3]

P.T.O.

SECTION "D"
[4Q. × 4 = 16 marks]

4.

a. Evaluate the Wronskian for the set of functions $\{e^{2x}, xe^{2x}\}$. [1]

b. Solve the second order differential equation

$$y'' + 2y' + y = 2x \sin x.$$

by the method of undetermined coefficients. [3]

5.

a. State Euler-Cauchy form of second order differential equation. [1]

b. Solve the differential equation $(x^2D^2 - 3xD + 3I)y = e^{2x}$. [3]

6. Consider the partial differential equation (PDE)

$$u_{xx} + y_{yy} = 0$$

a. Determine which type of PDE is this? [1]

b. Reduce the PDE into the canonical form. (Show your answer in detail). [3]

7. Using D'Alembert's method, find the solution $u = u(x,t)$ of the one-dimensional wave equation $u_{tt} = c^2 u_{xx}$ with initial conditions $u(x,0) = f(x)$ and $u_t(t,0) = g(x)$, and boundary conditions $u(0,t) = 0, u(L,t) = 0$ for all $t \geq 0$. (Show your work in detail.) [4]

SECTION "E"
[3Q. × 2 = 6 marks]

8. Solve the differential equation $(1 - x^2)\frac{dy}{dx} - xy = 1$. [2]

9. Solve the following system of first order ODEs by elimination method.

$$y_1' + 3y_2 = 0; \qquad y_2' - 2y_1 = 0$$ [2]

10. State *Convolution Theorem* and show that the convolution of $f(t) = e^t$ and $g(t) = e^{-t}$ is $\sinh t$. [1+1]