KATHMANDU UNIVERSITY
End Semester Examination
June/July 2024

Level	: B.Tech.	Course	: AIMA 201
Year	: II	Semester	: I
Time	: 2 hrs. 30mins.	F. M.	: 40

16 JUL 2024

SECTION "C"
[2 Q. × 6 = 12 marks]

1. What do you understand by integrating factor for the differential equation? Test the exactness of the following initial value problem:

$$2\sin(y^2)\,dx + xy\cos(y^2)\,dy = 0,\ \ y(2) = \sqrt{\frac{\pi}{2}}$$

If it is not exact, use the integrating factor to make it exact and solve. [1+1+2+2=6]

2. Define complementary function. Find the complementary function and particular integral of the differential equation:

$$y'' + 2y' + 101y = 10.4e^{x},\ \ y(0) = 1.1,\qquad y'(0) = -0.9$$

Also, find the general and specific solutions. . [1+1+2+1+1=6]

OR

Show that the integrating factor of linear differential equation: [3+3=6]

$$y'' + p(x)y' + q(x)y = r(x)$$

is $e^{\int p(x)dx}$. Solve the first order non-linear (Bernoulli) ODE IVP:

$$y' + (x+1)y = e^{x^2}y^3, y(0) = 0.5$$

SECTION "D"
[5Q. × 4 = 20 marks]

3. Solve the following system of differential equations by matrix method: [4]

$$\frac{dy}{dt} = x + 5y$$
$$\frac{dx}{dt} = -x + 3y$$

4. Define Laplace transform. Use it to find the solution of initial value problem: [1+3]

$$y'' + 2y' + y = e^{-t},\ \ y(0) = -1,\qquad y'(0) = 1$$

5. Define linear differential equation of first order. Solve the following differential equation: [1+3]

$$\frac{dy}{dt} + (\ln x)y = e^{-x}$$

6. Define Wronskian. Let p, q, r be constants on an open interval I, and the functions y_1 and y_2 form a basis of homogeneous equation corresponding to non-homogeneous equation

$$y'' + p(x)y' + q(x)y = r(x)$$

Show that the particular solution is $y_p = -y_1\int\frac{y_2 r}{w}dx + y_2\int\frac{y_1 r}{w}dx$ where w is the Wronskian of y_1 and y_2. [1+3]

P.T.O.

7. Find the solution $u(x, y)$ of the partial differential equation: [4]

$$u_x + u_y = (x + y)u$$

by separating the variables.

OR

Classify the following equations and solve by reducing to canonical form: [4]

$$u_{xx} + 4u_{xy} + 4u_{yy} = 0$$

SECTION "E"

[4Q. × 2 = 8 marks]

8. Find the general solution of Euler-Cauchy equation $x^2y'' - 4xy' + 6y = 0$.

9. Show that Laplace transform is linear.

10. Find the orthogonal trajectories of $xy = c$.

11. Solve:

$$y_1' = y_2$$
$$y_2' = y_1$$

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Marks Scored:

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Exam Roll No. :

Course : AIMA 201
Semester : I
F. M. : 10

Time: 30 mins.

Registration No.:

Date :

SECTION "A"

[20Q. × 0.5 = 10 marks]

Fill in the blank space (s) by writing the most appropriate word(s) or symbol(s).

1. The order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^3 + y = 0$ are ________________ and ________________ respectively.
2. The solution obtained from the general solution by giving a particular value to the arbitrary constants is called ________________________ solution.
3. The number of arbitrary constants in the general solution of differential equation of second order is ________________________.
4. ________________________ principle states that the linear combination of the solution of a homogeneous differential equation in an interval I is also the solution of that differential equation.
5. The solution of the partial differential equation of the form $Au_{xx} + 2Bu_{xy} + Cu_{yy} = F(x, y, u, u_x, u_y)$ is hyperbolic if ________________________.
6. Laplace transform of Dirac's delta function is ________________.
7. The differential equation $\frac{dy}{dx} + py = q$ is linear if p and q are functions of x alone or ________________________.
8. If the roots of the characteristic equation of a linear homogeneous differential equation are $1, 2 \pm \sqrt{3}i$, then the general solution is ________________________.
9. The convolution of t and 1 is ________________________.
10. The Wronskian of $y_1 = 1, y_2 = x$ and $y_3 = \frac{x^2}{2}$ is ________________________.

SECTION "B"

[20Q. × 0.5 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The condition for the exactness of the differential equation $Mdx + Ndy = 0$ is ________________________.

[$M = N$; $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$; $\frac{\partial N}{\partial y} = \frac{\partial M}{\partial x}$; $M = \frac{\partial N}{\partial x}$]

12. When the method of undetermined coefficient is used, the correct form of y_p for $y'' + 4y = 4e^{2x}$ is ______________________.
[Ae^x; Ae^{2x}; Ae^{Bx}; $4e^{2x}$]

13. The Euler-Cauchy equation $x^2y'' + a_1xy' + a_2y = f(x)$ can be transformed into linear differential equation by substituting ______________________.
[$x = \ln z$; $x = e^z$; $x = z^2$; $x = e^{z^2}$]

14. Let y_1 and y_2 be any two solutions of ordinary differential equation $y'' + Py' + Qy = 0$ and W be the corresponding Wronskian. Then which of the following is always true?
[If y_1 and y_2 are linearly dependent then there exist x_1, x_2 such that $W(x_1) = 0$ and $W(x_2) = 0$;
If y_1 and y_2 are linearly independent, then $W(x) = 0$ for every x;
If y_1 and y_2 are linearly dependent, then $W(x) \neq 0$ for every x;
If y_1 and y_2 are linearly independent, then $W(x) \neq 0$ for every x]

15. The partial differential equation $\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), c \neq 0$ is known as ______________________.
[heat equation; wave equation; Poisson's equation; Laplace equation]

16. A differential equation is considered to be partial differential equation if it has______________________.
[one dependent variable; more than one dependent variables; one independent variable; more than one independent variables]

17. The Laplace transform of Dirac delta function $\delta(t-2)$ is ______________________.
[$\frac{e^{-2s}}{s}$; e^{-2s}; e^{s-2}; $\frac{e^{-2s}}{s^2}$]

18. If $L\{f(t)\} = F(s)$, then $L\{t^n f(t)\} =$ ______________________.
[$-\frac{dF(s)}{ds}$; $(-1)^n\frac{dF(s)}{ds}$; $-\frac{d^nF(s)}{ds^n}$; $(-1)^n\frac{d^nF(s)}{ds^n}$]

19. The linear partial differential equations can be reduced into ordinary differential equations in which of the following method? ______________________.
[change of variable; fundamental equation; superposition principle; separation of variables]

20. The solution $u(x,y)$ of a PDE $u_{xx} - u = 0$ is______________________
[$f(y)e^x + g(x)e^{-x}$; $f(y)e^x + g(x)e^{-y}$; $f(y)e^x + g(y)e^{-x}$; $(f(y) + g(x))e^x$]