

KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

Marks Scored:

Level : B.Tech.

Year : II

Exam Roll No. :

Time: 30 mins.

Registration No.:

Course : AIMA 201

Semester : I

F. M. : 10

Date : 26 June

SECTION "A"

[10Q. \times 0.5 = 5 marks]

Fill in the blank space (s) by writing the most appropriate word(s) or symbol(s).

1. If $y = e^{2x}$ is a solution of $y'' - 5y' + ky = 0$, then the value of k is _____.
2. If an non-exact differential equation is multiplied by a function $F(x, y)$ and makes this differential equation exact, then the function $F(x, y)$ is called an _____.
3. The order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}} = \frac{dy}{dx} + xy$ is _____.
4. The integrating factor of a linear differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is _____.
5. The inverse Laplace transform of $\frac{1}{s}$ is _____.
6. The Laplace transform of e^{at} is _____.
7. The equation $\frac{\partial u}{\partial t} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$ is known as _____.
8. The differential equation $y'' + py + q = r$ is non-homogeneous, if _____.
9. The Wronskian of $y_1 = 1, y_2 = x$ and $y_3 = \frac{x^3}{3}$ is _____.
10. The convolution of two functions f and g is $(f * g)(t) =$ _____.

SECTION "B"

[10Q. \times 0.5 = 5 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The Laplace transform of Dirac delta function $\delta(t - a)$ is _____.
[$\frac{e^{-as}}{s}$; e^{-as} ; e^{s-a} ; $\frac{e^{-as}}{s^2}$]

12. The differential equation $u_{xx} + 4u_{xy} + 4u_{yy} = 0$ is _____.
 [elliptic; parabolic; hyperbolic; circular]
13. A partial differential equation $F\left(t, x, \dots, \theta, \frac{\partial \theta}{\partial t}, \frac{\partial \theta}{\partial x}, \dots, \frac{\partial^2 \theta}{\partial t^2}, \dots, \frac{\partial^2 \theta}{\partial x \partial t}, \dots\right) = 0$ is _____, if F is non-linear with respect to dependent variable but linear with respect to all its derivatives.
 [semi-linear; quasi-linear; fully non-linear; linear]
14. Two linearly independent solutions of ordinary differential equation $y'' + 6y' + 9y = 0$ are _____.
 [e^{3x} and xe^{3x} ; e^{3x} and $\ln e^{3x}$; e^{-3x} and xe^{-3x} ; e^{-3x} and $\ln e^{-3x}$]
15. The differential equation of the form $\frac{dy}{dx} + py = qy^n, n \in \mathbb{R}$ is a Bernoulli's differential equation only if _____ alone.
 [p is a function of x ; q is a function of x ;
 p and q both are functions of x ; p and q both are not functions of x]
16. A differential equation is considered to be ordinary if it has _____.
 [one dependent variable; more than one dependent variables;
 one independent variable; more than one independent variables]
17. If the roots of auxiliary equation of a second order non-homogeneous differential equation with constant coefficients are $2 \pm \sqrt{3}$, then the complementary function is _____.
 [$e^{2x}(A \cos \sqrt{3}x + B \sin \sqrt{3}x)$; $Ae^{-2x} + Be^{-\sqrt{3}x}$;
 $Ae^{2x} + Be^{\sqrt{3}x}$; $e^{-2x}(A \cos \sqrt{3}x + B \sin \sqrt{3}x)$]
18. Let $f(t)$ be a function with $f(0) = 0$ and Laplace transform of its derivative $L\{f'(t)\} = \frac{1}{s}$, then Laplace transform $L\{f(t)\} =$ _____.
 [$\frac{1}{s^2}$; $\frac{1}{s}$; $-\frac{1}{s^2+1}$; $\frac{s}{s^2+1}$]
19. The particular integral of $\frac{d^2y}{dx^2} - 4y = 1$ is _____.
 [$-\frac{1}{6}$; $-\frac{1}{5}$; $-\frac{1}{4}$; $\frac{1}{4}$]
20. The solution $u(x, y)$ of a PDE $u_{xx} - u = 0$ is _____.
 [$f(y)e^x + g(x)e^{-x}$; $f(y)e^x + g(x)e^{-y}$;
 $f(y)e^x + g(y)e^{-x}$; $(f(y) + g(x))e^x$]

KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

26-june

Level : B.Tech.
Year : II
Time : 2 hrs. 30 mins.

Course : AIMA 201
Semester : I
F.M. : 40

SECTION "C"

[2Q. × 6 = 12 marks]

1. What do you understand by first order linear differential equation? How is it different from Bernoulli's differential equation? Solve Bernoulli's differential equation: $\frac{dy}{dx} + x^5y = x^5y^7$ by reducing into linear form. [1+1+4]

OR

Show that the integrating factor of linear differential equation

$$y'' + p(x)y' + q(x)y = r(x) \text{ is } e^{\int p(x)dx}.$$

Solve the first order non-linear (Bernoulli)

$$\text{ODE IVP: } y' + (x + 1)y = e^{x^2}y^3, y(0) = 0.5 \quad [3+3]$$

2. If $L\{f(t)\} = F(s)$, prove that $L\{f'(t)\} = sF(s) - f(0)$. Use this notion to solve the IVP: $y'' - y' - 2y = 2t$, $y(0) = 1, y'(0) = 0$. [2+4]

SECTION "D"

[5Q. × 4 = 20 marks]

3. Use method of undetermined coefficients to find the general solution of the differential equation:

$$y'' - y' - 2y = 3e^{2x}$$

4. Solve the differential equation reducing into exact form.

$$(3xy - y^2)dx + x(x - y)dy = 0$$

5. Solve the following system of linear differential equations using eigenvalue method:

$$\begin{aligned} x_1' &= x_1 + 2x_2 \\ x_2' &= 2x_1 - 2x_2 \end{aligned}$$

with $x_1(0) = 1, x_2(0) = 0$
where $' \equiv \frac{d}{dt}$

OR

Define linear system of homogenous equations. Solve the following system of linear equations using elimination method.

$$\begin{aligned} y_1' &= 4y_1 - 2y_2 \\ y_2' &= 13y_1 - 6y_2 \end{aligned}$$

where $' \equiv \frac{d}{dt}$

6. Solve the PDE as ODE: $2u_{xx} + 9u_x + 4u = -3 \cos x - 29 \sin x$
7. Define unit step function and Dirac delta function. Find the Laplace transform of $u(t - 5) * e^{-3t}$

SECTION "E"
[4Q. \times 2 = 8 marks]

8. Find the orthogonal trajectories of $y = cx^{\frac{3}{2}}$.
9. Solve: $x^2y'' - 3xy' + 4y = 0$
10. Find the Laplace inverse of $F(s) = \frac{1}{(s-1)(s+1)}$
11. Use first shifting theorem to find the inverse Laplace transform of $\frac{1}{(s-1)^2}$.